Statistical modelling of strain-life fatigue data: prior distributions based on estimation of cyclic properties

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The correlation between the cyclic strain-life parameters of the Basquin-Manson-Coffin relation and the monotonic strength properties of steels attracted the interest of researchers in the past decades. Their use is very convenient prior to the design stage, e.g. for material selection, since it allows a simple estimation of strain-life curves. If tests are performed and their results are analyzed according to standards, these correlations are of no use. Instead, the information contained in these correlations can be used as prior knowledge in a Bayesian framework if it is accurately formalized as a probability function, potentially leading to a reduction of the number of tests to be executed.

The present article formulates a probabilistic strain-life model based on the Basquin-Manson-Coffin relation and presents a statistical re-evaluation of two of the correlations between monotonic and cyclic strength properties proposed in the literature. Clear distinction is made between the intrinsic variability of the fatigue life and the epistemic uncertainty due to the number of observations. A joint multivariate probability density function of the cyclic strength parameters is formulated including both sources of uncertainty, which can be used in Bayesian analysis.

1 Introduction

The strain-life curve of a material under uniaxial constant amplitude loading is commonly studied to evaluate the fatigue resistance of such a material. The number of cycles to fatigue crack initiation is related to the strain amplitude, expressed by the Basquin-Manson-Coffin relation [1–3], covering both the low and the high cycle fatigue regimes:

$$\varepsilon_a = \frac{\sigma'_f}{E}(2N_i)^b + \varepsilon'_f(2N_i)^c \tag{1}$$

where N_i is the number of cycles to fatigue crack initiation, σ'_f is the fatigue strength coefficient, b is the fatigue strength exponent, ε'_f is the fatigue ductility coefficient, c is the fatigue ductility exponent, and ε_a is the cyclic strain amplitude. Figure 1 shows typical strain to cycle curves, the exponents b and c represent the slopes of the elastic and plastic strain vs. cycle curves respectively, whereas the coefficients σ'_f/E and ε'_f correspond to the elastic and the plastic strain amplitude for a single load reversal, i.e. half a cycle, respectively. This representation is also useful to distinguish the low cycle fatigue regime, which is characterized by high plastic strain amplitude, and the high cycle fatigue regime, which is characterized by low plastic strain amplitudes. For the purpose of this paper, σ'_f, b, ε'_f, c are referred to as cyclic strain-life parameters. According to current standards [4], their determination involves a linear regression analysis of fatigue test data using the least square method.

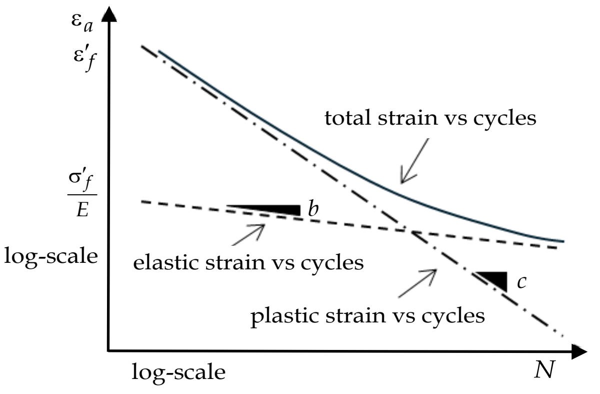

Figure 1. Strain vs cycles curves

Many researchers focused on the relation between the cyclic strength properties to other material properties, such as the ultimate tensile strength, yield stress, or Brinell hardness, specifying that these relations do not aim to be an alternative to fatigue testing, but can be used in a preliminary stage of the design to select the material and to study the effect of the geometry, depending on the experience of the designer. Among those, the correlations proposed by Roessle and Fatemi [6] and the median estimation proposed by Meggiolaro [5], gave the best agreement with test data. Meggiolaro [5] performed an extensive statistical evaluation of the cyclic strength parameters involving more than 800 metals. It was concluded that

1. The exponents *b* and *c* are independent of the strength of the metallic alloy but they do depend on its family, e.g. steel, aluminium, or titanium.
2. The fatigue strength coefficient is strongly correlated to the ultimate tensile stress or the Brinell Hardness.
3. The fatigue ductility coefficient does not correlate well with monotonic tensile test properties, and not even with the true fracture ductility. They suggest estimating the fatigue ductility coefficient based on a constant value for each alloy family.

Roessle and Fatemi [6] reach similar conclusions for points (1) and (2), whereas a relation is proposed between the Brinell Hardness and the fatigue ductility coefficient ε'_f .

The correlations are obtained by relating for each dataset the expected value of each cyclic strength property and the measured value of the Brinell hardness. Following the analysis of several datasets, a probability density function can be estimated for each cyclic strength parameter. Such a probability density function was proposed in [5], however, when derived in this way it only accounts for the batch-to-batch variability of the cyclic strength parameters. Two main drawbacks are individuated as a result of this procedure:

1. the epistemic uncertainty is not taken into account. This is because for each dataset only the estimator of each cyclic strain-life parameter is used, and the number of test data is unknown. This biases the estimation of the variability, since small datasets are considered to have the same relevance as larger ones, for which it is more likely that the estimator of a cyclic strain-life parameter is closer to the real value.
2. Some datasets contain data obtained at large strain ranges, despite it is possible to determine all the cyclic strength parameters, an extrapolation to longer lives is largely affected by errors.

As indicated in [4] these correlations should never be used for design, therefore fatigue testing is still necessary. Fatigue testing remains a relatively expensive task: it is required to obtain at least 15 - 30 experimental data to characterize the relation between stress/strain amplitude and the fatigue life [7] with a sufficient accuracy. As a result of testing, the observed random realizations of the fatigue life lead to the estimation of the physical variability associated with fatigue, however another source of uncertainty is also present and it is related to the number of observations, i.e. of test data available, and it is referred to as epistemic uncertainty or sampling error [8]. Whereas the observed intrinsic variability, i.e. the aleatory uncertainty stabilizes with increasing the number of experimental observations, the epistemic uncertainty always reduces, and it can be quantified by estimating interval or distribution estimators, such as the width of a confidence interval, e.g. 95%, of the mean value or a tolerance interval on lower/upper bound predictions. These prediction bounds also account for the aleatory uncertainty. Another way to quantify the epistemic uncertainty is to determine the standard error of the parameters of the statistical model used to infer the experimental data. For relatively small fatigue test datasets, the sampling error associated to the estimator can be significantly larger than the aleatory uncertainty [9]. In [10] the authors made use of several statistical techniques to estimate the statistical uncertainty and incorporated them in the estimation of the 5% lower prediction bound of the fatigue life using a Monte Carlo procedure, as described in [11].

Bayesian inference has been used with the aim of reducing the uncertainty and to account for prior knowledge [12]. Different from frequentist inference, Bayesian inference can result in useful results even if the dataset contains a relatively small amount of test data due to the fact that prior knowledge is used [13]. By using Bayesian inference, the reliability of the estimation can potentially increase if technological knowledge from theoretical studies or previous test data is accurately formalised by the prior distributions. A Bayesian methodology was formulated to produce design S-N curves based on small censored datasets, giving consistent results with frequentist methods in [14]. Statistical uncertainty, i.e. the sampling error, was accounted for as well as runout data. A Bayesian framework was presented in [15], where several fatigue datasets on welded connections transversally loaded were used to define a prior distribution of the negative inverse slope of the Basquin relation. Bayesian analyses were also conducted for determination of S-N curves of metallic fatigue data [16], using informative and an empirical estimation of the prior. More background information about these methods can be found in [17].

Despite the wide range of applicability of the strain-life method, few statistical models have been formulated for the inference of ε_a - N_i data. Based on the indications given in the ASTM standard E739 [4], a method of deriving the cyclic strength properties from experimental data is given in [18]. However, this method does not explicitly account for a probability function representing the variability of the fatigue life. A regression model for strain-life data was presented in [19] as an alternative to the Basquin-Manson-Coffin equation. A statistical model based on Equation (1) was presented in [20]. In this model, the heteroscedasticity of fatigue life data is accounted for by assuming the log-fatigue life to be distributed according to a Normal probability density function and by assuming the standard deviation to be linearly related to the strain range. The aim is to model an increasing scatter with decreasing stress range.

This work presents an analysis of strain-life fatigue test data aimed to quantify the epistemic uncertainty underlying the correlations between cyclic and monotonic strength properties. These correlations are corroborated by considering multiple fatigue datasets related to unalloyed and low alloyed steels, similarly to what was done in [5]. The main difference consists in the method of estimating the cyclic strength properties. In the present work this is corroborated by the estimation of the epistemic uncertainty. In order to do so, a statistical model for strain-life fatigue data is formulated accordingly to [4] and Equation (1). The selected datasets are inferred and the static strength properties and their uncertainty are estimated. A joint distribution of the cyclic strength parameters is formulated as a result of both the correlation with the Brinell Hardness proposed by Roessle and Fatemi [6] and the median method proposed by Meggiolaro [5]. Differently from other studies, the relevance of each dataset is also taken into account, which is proportional to the number of experimental observations contained in it. In other words, relatively small datasets are not considered to have the same relevance as larger ones. The estimation of the epistemic uncertainty is made by using the maximum likelihood method, resulting in a joint distribution that can be used as a prior distribution for the inference of fatigue test data. The benefit consists in the possibility of using smaller datasets for experimental analysis of steels of the same alloy family.

2 Models and Methods

In the present section the models and the methods used to analyze the data are formulated and presented with application to strain life data.

*2.1 **Statistical estimation of the cyclic strength parameters and their uncertainty***

The statistical model used in this work is based on Equation (1), which expresses the total strain amplitude as function of the initiation life given the model parameters, i.e., the cyclic strength properties. When fatigue test data are inferred, the independent variable should be the stress/strain amplitude (range) and the number of cycles at the test termination, whether it is a failure or a runout, should be set as the dependent one [21,22]. According to the current standard, the measured strain amplitude is divided in the elastic and the plastic components $\varepsilon_{a,e}$ and $\varepsilon_{a,p}$, respectively:

$$\varepsilon_{a,e} = \frac{\sigma_a}{E} \tag{2}$$

$$\varepsilon_{a,p} = \varepsilon_a - \varepsilon_{a,e} \tag{3}$$

where σ_a is the stress amplitude, and E is Young's modulus. It follows that two independent inferences have to be carried out, one to relate the logarithm of the elastic strain component to the logarithm of the number of reversals to failure, and one to relate the logarithm of the plastic strain component to the logarithm of the number of reversals to failure. If $x = \log_{10}(\varepsilon_a)$ and $w = \log_{10}(2N_i)$ the median curve is determined by:

$$w = Ax + B + \varepsilon(0, s) \tag{4}$$

the error ε is assumed to be distributed according to a Normal distribution with 0 mean and standard deviation equal to s, meaning that the fatigue life to crack initiation N_i is assumed to be distributed according to a Lognormal distribution. The parameters of the linear regression model, namely A, B and s, need to be estimated in either a frequentist or a Bayesian statistical framework.

2.1.1 *Likelihood function and maximum likelihood estimation*

The maximum likelihood method [23] is based on the use of the likelihood function to estimate the maximum likelihood estimator (MLE) of the parameters of the regression model, assuming a probability function to model the fatigue life. The parameters of the linear regression model are referred to as $\theta = \{A, B, s\}$. Therefore, the probability density function (PDF) of the fatigue life can be noted as $\phi_W(w; \varepsilon_a, \theta)$. The procedure involves the estimation of the loglikelihood function for failure and runout data [23, 24]:

$$L(\theta; Data) = \sum_{j=1}^{n} [\phi_W(w_j; \varepsilon_{a,j}, \theta)]^{\delta_j} [1 - \Phi_W(w_j; \varepsilon_{a,j}, \theta)]^{1-\delta_j} \tag{5}$$

where $L(\theta; Data)$ is the loglikelihood that is a function of the model parameters given the dataset, the index j denotes the generic datapoint, ϕ_W is the cumulative distribution function of the fatigue life, and δ is the failure indicator ($\delta = 1$ for failure, and $\delta = 0$ for runout). The value of θ that maximizes the loglikelihood function is the maximum likelihood estimator θ_{MLE}. The epistemic uncertainty is expressed in terms of sampling distribution. It represents the probability that a statistic of interest, i.e., a parameter of the model, would assume a certain value if the sample of the same size was collected several times from the population. The sampling distribution can be estimated using the loglikelihood function. The Fisher Information matrix of the likelihood function, which is estimated as the negative Hessian matrix of the loglikelihood, is directly related to the epistemic uncertainty [23], meaning that the sampling distribution of θ is assumed to be a multivariate normal distribution:

$$\theta \sim MVN\left(\theta_{MLE}, I^{-1}(\theta)\right) \tag{6}$$

where $I^{-1}(\theta)$ is the inverse of the Fisher information matrix, and MVN denotes a multivariate Normal distribution. It follows that width of the sampling distribution is related to the 'peakness' of the loglikelihood function. A flatter likelihood function at the MLE results in more uncertainty, and vice-versa.

Equation 5 is applied to each individual dataset, i.e. per material and literature source, to obtain the cyclic strain-life parameters. This allows to obtain a maximum likelihood estimator and an observed Fisher information matrix for each dataset.

A transformation is necessary to evaluate either the cyclic coefficients or the exponents of Equation (1). For example, when fitting the plastic strain vs number of cycles to obtain the fatigue ductility exponent and coefficient, the aforementioned procedure results into A_p, B_p and s_p hence:

$$c = \frac{1}{A_p} \tag{7}$$

$$\varepsilon'_f = 10^{-cB_p} \tag{8}$$

The fatigue strength exponent and coefficient are calculated in the same way as the ductility parameters when fitting elastic strain vs number of cycles data. However, to use Equations (7) and (8), the base 10 logarithm of the stress amplitude should be used as independent variable x in Equation (4) instead of the strain amplitude.

3 Results and discussion

The results presented in this paper are produced considering 65 datasets of fatigue test data reported in [25-27]. This means that only a selection of data in these sources is used in this paper, because some datasets cannot be considered equivalent in terms of sample or test conditions, or because of incomplete information available for the evaluation. Therefore, the selection is made on the following bases:

1. The surface condition is polished.
2. The tests have been performed using a zero mean stress.
3. The datasets contain more than 6 data.
4. The ultimate tensile strength of the material has been tested from the same batch of material.
5. The test temperature is below 400 °C.

Surface condition is a well-known influencing factor of the fatigue strength of materials, when fatigue cracks initiate at the free surface. Any other surface condition resulting in a higher roughness of the specimen surface would have a detrimental effect on the fatigue strength, affecting the cyclic strength parameters, therefore it is not considered for the purpose of this paper.

Some authors [25-27] corrected the test data obtained under a non-zero mean stress using a mean stress correction model before inferring the data using the Basquin-Manson-Coffin equation. However, when the cyclic strength parameters are obtained as a result of data that have been corrected with a mean stress correction model, their value would be affected by the uncertainty and the accuracy of the model selected for this purpose. Datasets with less than 6 data have been disregarded due to the low statistical significance associated to the estimated cyclic strength parameters and the standard deviation of the number of cycles to failure. Being five the parameters to be estimated, one would need at least six data to do so.

Also, datasets for which the ultimate tensile strength has not been reported have been disregarded since they do not allow the correlation between the monotonic tensile property and the cyclic strength parameters. The reason why the test data at a temperature above 400 °Celsius have not been considered is that for these datasets the ultimate tensile strength has not been measured at room temperature. Although, as stated by Meggiolaro in [5], the cyclic strength parameters are not largely affected by the testing temperature, the ultimate tensile strength does depend on it for temperatures above 400 Celsius [28-29].

Each remaining dataset contains fatigue test data, for which the number of cycles to failure is related to the total strain amplitude and the stress amplitude, and monotonic strength properties.

In the following sections, two options are provided to obtain a prior distribution on the basis of the comparison between the two considered correlations and the considered datasets. In one case, the prior distribution is obtained from the distribution of the residuals for the parameter of interest. In the second case, it is obtained as a marginal distribution.

3.1 ***Case 1. Results of the inference***

The MLE of cyclic strength parameters resulting from the inference of the selected datasets are reported in Figure 2, where a comparison is made with the median trend of the correlations proposed by Roessle and Fatemi [6] and Meggiolaro [5]. For clarity, the datasets are divided per source. It should be noted that Roessle and Fatemi derived their correlation using both the Brinell hardness and the ultimate tensile strength. The latter version is used here for easier comparison between the two correlations and the results on the considered datasets.

To estimate the uncertainty, the distribution of the residuals are obtained for each dataset:

$$\varepsilon(0, s_z) = \hat{z} - z \tag{9}$$

where $\hat{z}$ represents the estimate of the generic parameter resulting from the inference of the data, z results from either the correlation of either Roessle and Fatemi [6] or the Median method [5]. A bias type of error resulting from a non-optimal fit is considered into a larger standard deviation of the residuals s_z .

The presence of a bias is revealed by the median value (CDF = 0.5) not being equal to zero. A smaller bias is encountered using the correlation of Roessle and Fatemi. Nevertheless, the standard deviations of the residuals are practically coincident for the two methods.

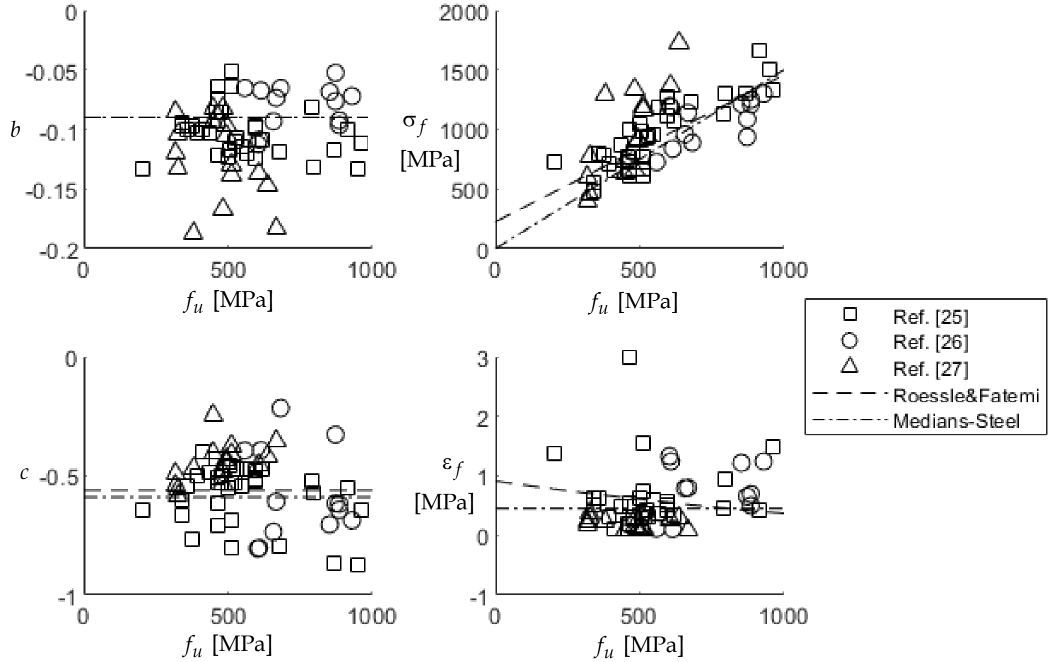

Figure 2. Comparison of strain-life properties for the different datasets and existing correlations of Roessle and Fatemi [6] and Medians [5] as a function of the ultimate tensile stress

The prior distribution for a cyclic strength parameter is formulated as a student t distribution shifted by the value of the cyclic strength parameter estimated by the correlation, and scaled considering the standard deviation of the residuals and the difference between the ultimate tensile strength for the material at hand and the ultimate tensile strength of the materials considered in this study. A closed form solution is non-trivial, however given a certain material characterized by its ultimate tensile strength f_u , a value of the prior distribution can be sampled considering the following equation:

$$z(f_u) + t_{p,n-1}^{-1}\, s_z \sqrt{1 + \frac{1}{n} + \frac{(f_u - \overline{f}_u)^2}{\sum (f_{ui} - \overline{f}_u)^2}} \tag{10}$$

Where $z(f_u)$ is the value of the parameter of interest estimated through one of the two correlations, $t_{p,n-1}^{-1}$ is the inverse CDF of the T-student distribution related to a random probability p and with n – 1 degrees of freedom, where n = 65 is the number of considered datasets, s_z is the standard deviation of the residuals, $\overline{f}_u$ is the average value of the

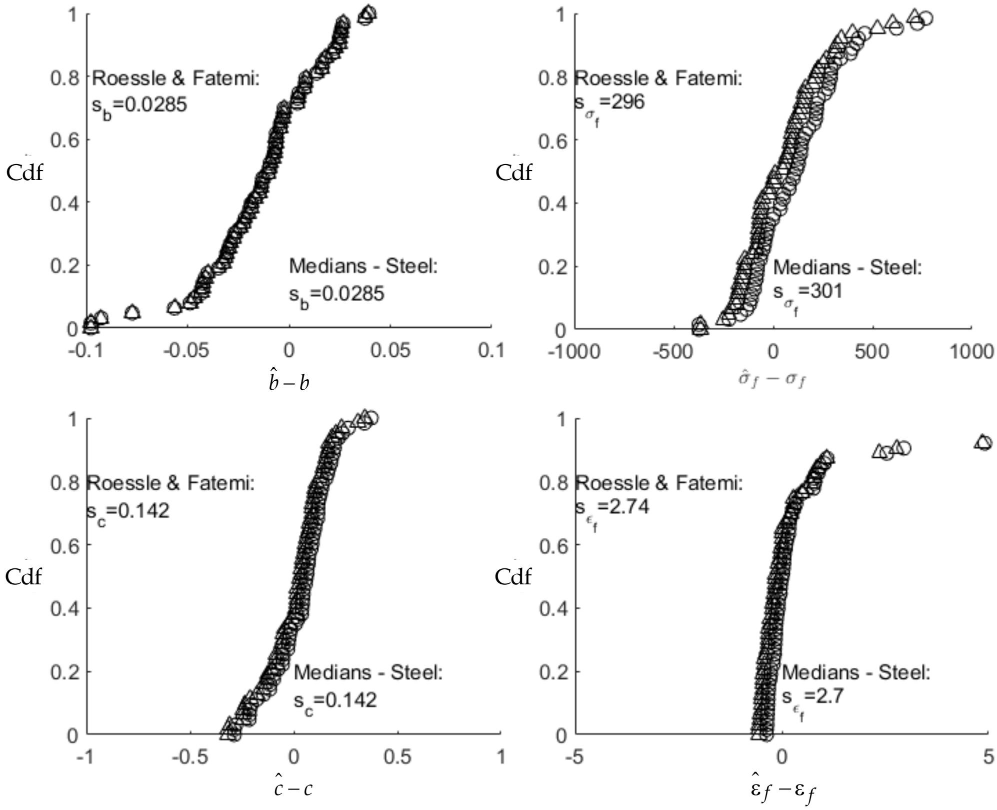

Figure 3. Distribution of the residuals of the cyclic parameters compared to Roessle and Fatemi [6] and Medians [5]

ultimate tensile strength of the collection of datasets, i.e. the abscissa of the centroid of the data, and f_{ui} is the ultimate tensile strength for each dataset. Considering the datasets in this study $\overline{f}_u$ = 568.9 MPa, and $\sum(f_{ui} - \overline{f}_u)^2$ = 2.8 10[6]. The distributions of the residuals are visualized in Figure 3, where the value of s_z is also reported for each cyclic parameter. This way, as it happens for a prediction interval for a linear regression problem, the prior distribution also considers epistemic uncertainty, by means of the T-Student distribution, and widens with increasing the absolute difference between f_u and $\overline{f}_u$.

3.2 ***Case 2. Marginal Prior***

Different than in the previous section, here the prior distribution is generated in terms of the parameters *A* and *B* of the linear regression model, see Equation (4), which are related to the cyclic strength parameters by Equations (7) and (8) and their equivalents for fatigue strength exponent and coefficient. This is done because in the MLM the fitting parameters are assumed distributed according to a MVN, therefore the distribution of the cyclic strength parameters depend on the transformation resulting from the application of Equations (7) and (8). In this case, the derived prior is not related to a specific correlation.

Figure 4 shows the marginal distribution of the parameters of the regression model, namely A_{el} , B_{el} , A_{pl} and B_{pl} resulting from the inference of the 65 datasets. For each parameter, two marginal distributions have been plotted, one considering the aleatory uncertainty only and the other one considering both the aleatory and the epistemic

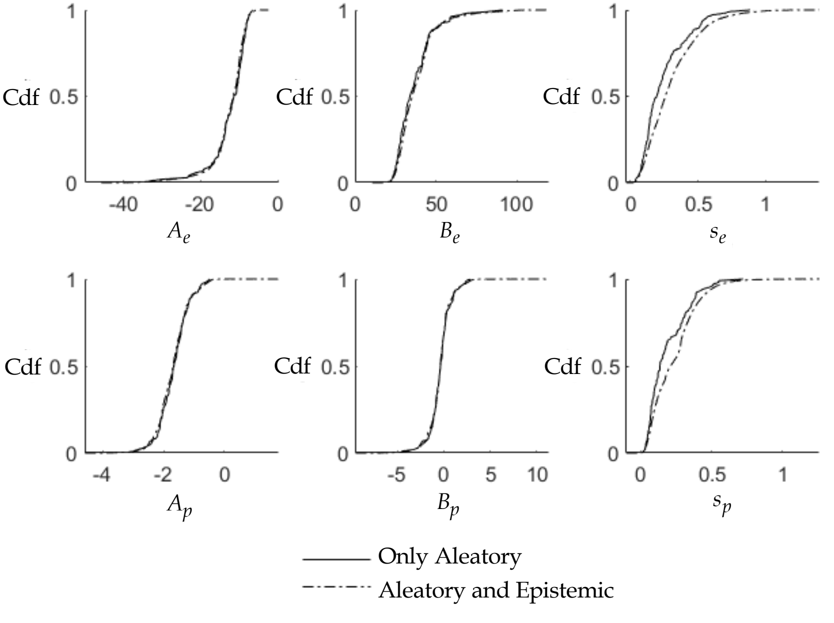

Figure 4. Marginal distributions of the fitting parameters for both the elastic and plastic strain vs life curves

uncertainties. It can be seen that the resulting distributions do not differ substantially, except that heavier tails result when both sources of uncertainty are considered. This is due to the large number of data contained in the datasets. The median values of the standard deviation for the log-fatigue life are 0.258 and 0.211 for the elastic strain versus life relation and for the plastic strain versus life relation. By assuming the two curves to be uncorrelated, this results in a total median value of 0.326. This assumption is justified by the poor correlation obtained between the parameters of the elastic strain vs. life and the plastic strain vs. life curves. Table 1 shows the mean, the median, the standard error and the correlation matrix estimated for the marginal distribution.

Table1. Marginal distribution of the parameters observed for the selected datasets

parameter	mean	median	standard error	correlation coefficient					
				A_e	B_e	s_e	A_p	B_p	s_p
A_e	-12.6	-11.6	4.46	1.000	-0.989	-0.304	0.125	0.150	-0.361
B_e	37.6	35.0	12.0		1.000	0.318	-0.065	-0.107	0.374
s_e	0.299	0.258	0.192			1.000	-0.350	-0.295	0.609
A_p	-1.69	-1.69	0.505				1.000	0.930	-0.140
B_p	-0.382	-0.377	1.21					1.000	-0.131
s_p	0.233	0.211	0.156						1.000

One of the advantages of this multivariate distribution is that it can be deployed as a prior distribution in a Bayesian framework either entirely or partially, depending on the specific circumstances. For example, only the information related to the scatter of some of the cyclic parameters can be used as prior information when the examined material is not within the population used to extract the considered samples, under the assumption that the scatter of these parameters is not dependent on the specific steel grade. Furthermore, this option is viable when the considered steel grade is characterized by an ultimate tensile strength beyond the bounds of the considered datasets, which are visible in Figure 2.

Another advantage is the estimation of the scatter of the fatigue life and its underlying distribution, which is an information not provided by neither of the correlations between static strength and cyclic properties.

4 Conclusions

This paper presents the derivation of statistical distributions of cyclic strain-life properties for several steels for which test data are obtained from literature, namely unalloyed, low-alloyed, and high alloyed. Strain life properties can either result from tests or be estimated from statistical correlations with other material properties, like tensile strength and hardness. Even though the estimate of cyclic properties from static ones is not to be intended as a replacement for the tests, it can be used for first design estimates. On the basis of a literature research, two correlations are considered here.

The comparison between strain life properties obtained from experimental data and the considered correlations allows the estimation of the uncertainty underlying the estimation of such properties. This highlights the inherent scatter of some of these properties, like the cyclic ductility and strength exponents, which can only be considered as constants while using the correlations. Moreover, also the knowledge about the strength and ductility coefficients are corroborated by means of the derivation of the conditional probability distribution. These distributions allow conducting more informed explorative studies at the design stage and are also useful for conducting sensitivity studies in which the effect of varying these parameters within reasonable bounds is required. Furthermore, a statistical estimation of the typical scatter of fatigue in low and high cycles regimes is provided in terms of standard deviation of the logarithm of the fatigue life.

In addition to this, a multivariate normal distribution of all cyclic parameters is formulated, which can be deployed as a weak, yet informative, prior distribution in a Bayesian inference framework, hence leveraging on these existing correlations for less uncertain designs.

Despite the sound statistical basis of the proposed prior distribution, the authors suggest its use in a explorative design stage. The use of such distribution as a prior for dimensioning or assessing a structural design is to be intended as explorative and extreme care should be paid by the users in assessing whether the prior distribution is adequate and does not induce statistical bias reflecting into a potentially unsafe design or assessment.

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