Discounting for optimal and acceptable technical facilities involving risks

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Technical facilities should be optimal with respect to benefits and cost. Optimization of technical facilities involving risks for human life and limb require an acceptability criterion and suitable discount rates both for the public and the operator depending on for whom the optimization is carried out. The *life quality index* is presented and embedded into modern socio-economic concepts. A general risk acceptability criterion is derived. The *societal life saving cost* (= implied cost of averting a fatality) to be used in optimization as live saving or compensation cost and the *societal willingness-to-pay* based on the *societal value of a statistical life* or on the *societal life quality index* are developed, the latter for three different mortality regimes. Discount rates γ must be long term averages in view of the time horizon of some 20 to more than 100 years for the facilities of interest and net of inflation and taxes. While the operator may use long term averages from the financial market for his cost-benefit analysis the assessment of interest rates for investments of the public into risk reduction is more difficult. The classical Ramsey model decomposes the real interest rate (= output growth rate) into the rate of time preference of consumption and the rate of economical growth multiplied by the elasticity of marginal utility of consumption. It is found that the rate of time preference of consumption should be a little larger than the long term population growth rate if used for the determination of parameters in the acceptability criterion. The output growth rate on the other hand should be smaller than the sum of the population growth rate and the long term growth rate of a national economy which is around 2% for most industrial countries. Accordingly, the rate of time preference of consumption is about 1%, which is also intergenerationally acceptable from an ethical point of view. It is also shown that given a certain output growth rate there is a corresponding maximum interest rate in order to maintain non-negativity of the objective function.

Key words: Optimum technical facilities, life quality index, risk acceptability, discounting

1 Optimal technical facilities

A technical facility is optimal if the following objective is maximized:

$$Z(\mathbf{p}) = B(\mathbf{p}) - C(\mathbf{p}) - D(\mathbf{p}) \tag{1}$$

For the purpose of this paper it is assumed that all quantities in eq. (1) can be measured in monetary units. $\mathbf{p}$ is the vector of all safety relevant parameters. $B(\mathbf{p})$ is the benefit derived from the existence of the facility, $C(\mathbf{p})$ is the cost of design and construction and $D(\mathbf{p})$ is the cost in case of failure. Statistical decision theory dictates that expected values are to be taken. In the following it is assumed that $B(\mathbf{p})$, $C(\mathbf{p})$ and $D(\mathbf{p})$ are differentiable in each component of $\mathbf{p}$ The cost may differ for the different parties involved and having economic objectives, e.g. the owner, the builder, the user and society. A facility makes sense only if $Z(\mathbf{p})$ is positive within certain parameter ranges for all parties involved.

The facility has to be optimized during design and construction at the decision point, i.e. at time $t = 0$. Therefore, all cost need to be discounted. A continuous discounting function, the discount factor, is assumed which is accurate enough for all practical purposes

$$\delta(t) = exp\left[-\int_0^t \gamma(\tau)d\tau\right] = exp\left[-\gamma t\right] \tag{2}$$

where $\gamma = \gamma\ (t)$ is the time-independent interest rate.

In general, one has to distinguish between two replacement strategies, one where *the facility is given up after service or failure* and one where the *facility is systematically replaced after failure*. Further, we distinguish between facilities which *fail upon completion or never* and facilities which *fail at a random point in time* much later due to service loads, extreme external disturbances or deterioration. The option failure upon completion or never implies that loads and resistances on the facility are time-invariant. Reconstruction times are assumed to be negligibly short.

For simplicity, the objective function is only derived for the special case of failure under external disturbances and systematic reconstruction. Assume random events in time forming a renewal process. The times between failure events are independent and have probability density $f(t), t \geq 0$. For constant benefit per time unit $b(t) = b$ and $f_n(t, \mathbf{p})$ the density of the time to the n-th renewal an objective function can be derived by making use of the convolution theorem for Laplace transforms. Laplace transforms are defined by $f^*(\gamma) = \int_0^\infty e^{-\gamma t} f(t)dt$ and there is $0 \leq f^*(\gamma) \leq 1$ if $f(t)$ is a probability density and $f^*(0) = 1$ and $f^*(\infty) = 0$. In the transformed space there is $h^*(\gamma) = f(\gamma)^* g^*(\gamma)$ for $h(t) = \int_0^\infty f(t-\tau)g(\tau)d\tau$, an operation necessary to determine $f_n(t)$. The simplicity of this operation is the main reason why a continuous discounting function is used. Direct discrete discounting is already considered in [13] by making use of generating functions. Then, one derives

$$\begin{aligned} Z(\mathbf{p}) &= \int_0^\infty be^{-\gamma t}dt - C(\mathbf{p}) - (C(\mathbf{p}) + H)\sum_{n=1}^{\infty}\int_0^\infty e^{-\gamma t} f_n(t, \mathbf{p})dt \\ &= \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{f^*(\gamma, \mathbf{p})}{1 - f^*(\gamma, \mathbf{p})} \quad = \frac{b}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)h^*(\gamma, \mathbf{p}) \end{aligned} \tag{3}$$

where $f^*(\gamma, \mathbf{p})$ is the Laplace transform of $f\ (t, \mathbf{p})$ and $h^*(\gamma, \mathbf{p})$ is the Laplace transform of the renewal density (renewal intensity) $h\ (t, \mathbf{p})$. H is the monetary loss in case of failure including direct failure cost, loss of business and, of course, the cost to reduce the risk to human life and limb. If, in particular, at an extreme Poissonian loading event (e.g. flood, wind storm, earthquake, explosion) failure occurs with probability $P_f(\mathbf{p})$ one obtains for independent failure events [15] [39] :

$$h^*(\gamma,\mathbf{p}) = \sum_0^\infty f^*(\gamma)\, P_f(\mathbf{p}) \left[f^*(\gamma)\, R_f(\mathbf{p})\right]^{n-1} = \frac{P_f(\mathbf{p})\, f^*(\gamma)}{1-R_f(\mathbf{p})\, f^*(\gamma)} = \frac{\lambda\, P_f(\mathbf{p})}{\gamma} \tag{4}$$

with $R_f(\mathbf{p}) = 1 - P_f(\mathbf{p})$ and $f^*(\gamma) = \dfrac{\lambda}{\gamma+\lambda}$ for $f(t) = \lambda \exp[-\lambda t]$. An important asymptotic result for arbitrary failure models is

$$\lim_{t\to\infty} h(t,\mathbf{p}) = \lim_{\gamma\to 0} \gamma\, h^*(\gamma,\mathbf{p}) = \frac{1}{E[T(\mathbf{p})]} \tag{5}$$

where $E[T(\mathbf{p})]$ is the mean time between renewal.

The precise details of this and more general renewal models can be found in [33] . Many other objective functions can be formulated. For example, serviceability failure, obsolescence, aging, deterioration and inspection and maintenance and finite service times can be dealt with (see [33] [34] [44]). Benefit and damage term can be functions of time [16] [45] [49] . Also, multiple mode failures (series systems) with stationary failure models or even non-stationary failure models can be considered [36] .

In accordance with economic theory benefits and (expected) cost should be discounted by the same rate as done above. Different parties, e.g. the owner, operator or the public, may, however, use different rates. While the owner or operator may take interest rates from the financial market the assessment of the interest rate for an optimization in the name of the public is difficult. The requirement that the objective function must be non-negative leads immediately to the conclusion that the interest rate must have an upper bound γ_{max} depending on the benefit rate $b = \beta C(\mathbf{p})$ (see [16]). For the model in eq. (4) we have

$$\frac{\beta C(\mathbf{p})}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H)\frac{\lambda P_f(\mathbf{p})}{\gamma} = 0 \tag{6}$$

and, therefore, by solving for γ and given (optimal) $\mathbf{p} = \mathbf{p}^*$

$$\gamma < \gamma_{max} < \beta - \lambda P_f(\mathbf{p})\left(1 + \frac{H}{C(\mathbf{p})}\right) \tag{7}$$

implying $\gamma < \beta$ for $\lambda P_f(\mathbf{p}) << \beta$. It follows that the benefit rate β must be slightly larger than γ_{max}. From eq. (6) one also concludes that there must be $\gamma > \mathbf{0}$ because the limit $\gamma \to 0^+$ is $\pm\infty$ or at least undefined. The quantification of public interest rates $\gamma_{max} < \beta$ will be discussed in detail in the paper. Any optimization of cost and benefits must include the cost to reduce the risk to human life and limb and, possibly, a criterion setting a limit which is generally acceptable. A new approach based on the so-called life quality index and health-oriented economics will be developed and discussed.

2 Rational socio-economically based risk acceptance criteria – the life quality index

The question of limiting the risks to human lives is essentially the question of how much society is willing to pay and can afford to *"reduce the probability of premature death by some intervention changing the behavior and/or technology of individuals or organizations"* [46] . Further, any argumentation must be within the framework of our moral and ethical principles as laid down in our constitutions and elsewhere including everyone's right to live, the right of a free development of her / his personality and the democratic equality principle. It is clear that only involuntary risks, i.e. risks to which an anonymous member of society is exposed involuntarily from its technical or natural environment, can reasonably be discussed here.

Cantril [9] and similar more recent studies conclude from empirical studies that long life and wealth are among the primary concerns of humans in a modern society. Life expectancy at birth (mean time from birth to death) e is the area under the survivor curve (survival function) $\ell(a) = exp\left[-\int_0^a \mu(t)dt\right]$, i.e.

$$e = e(0) = \int_0^{a_u} \ell(a)da \tag{8}$$

where a_u = largest age considered and $\mu(a)$ = age dependent mortality or force of mortality. Another suitable indicator of the quality of life is the gross domestic product (GDP) per capita and year. The GDP is roughly the sum of all incomes created by labor and capital (stored labor) in a country during a year. It provides the infrastructure of a country, its social structure, its cultural and educational offers, its ecological conditions among others but also the means for the individual enjoyment of life by consumption. In most developed countries about 60±5% of the GDP is used privately, 20±5% by the state (e.g. for military, police and jurisdiction) and the rest for investments. The GDP also creates the possibilities to "purchase" additional life years through better medical care, improved safety in road and railway traffic, more safety in or around building facilities, more safety from hazardous technical activities, more safety from natural hazards, etc.. It does not matter whether those investments into "life saving" are carried out individually, voluntarily or enforced by regulation or by the state via taxes. If it is assumed that neither the share for the state nor the investments into depreciating production means can be reduced, only the part for private use is available for risk reduction. Therefore, the part available for risk reduction is $g \approx 0.6$ GDP. The exact share for risk reduction must be determined separately for each country or group in a country.

In 1998 approximately 10% of the GDP were used for health care in industrialized countries [55]. Data for almost all other expenses for risk control and risk reduction, i.e. in road, railway and air traffic, in structural and fire safety, in protection against natural hazards, etc., are either absent or unreliable but some few more percent of the GDP are likely to be spent. Overall mortality (per year) is about 0.01 but only 3 in 10,000 are not due to natural causes. If one substracts from this number those deaths which are induced by voluntary risks (sports and some traffic accidents), then, the reduction of a mortality a little less than 0.0003 is the subject of our study.

Lind [22] sets out from a composite social indicator

$$L = L(a, b, \ldots, e, \ldots) \tag{9}$$

with $a, b, \ldots, e, \ldots$ certain social indicators. Let it be differentiable so that:

$$dL = \frac{\partial L}{\partial a} da + \frac{\partial L}{\partial b} db + \ldots + \frac{\partial L}{\partial e} de + \ldots \tag{10}$$

If only the two factors mentioned before, that is g and e, are considered dL vanishes for

$$dL = 0 \Rightarrow \frac{dg}{de} = -\frac{\frac{\partial L}{\partial e}}{\frac{\partial L}{\partial g}} \tag{11}$$

implying that a change in e should be compensated for by an appropriate change in g. Any investment (reduction by dg) into life saving must be compensated by a gain de in life expectancy so that L remains unchanged or vice versa. Assume that L is the product of a function of g (as a measure of the quality of life) and another function of the time $t = (1 - w)e$ to enjoy life (as a measure of the quantity of life) where w is the time to be spent in paid work. The individual can now increase leisure time by either increasing life expectancy by risk reduction or by reducing the time spent in economic production which generally means smaller income. Assume then that the quantity w is chosen such that L is maximized. This appears to be a reasonable assumption because most work is dull, boring, troublesome and sometimes dangerous. One also can draw on a historical argument. In 1870 the yearly time spent in work was 2900 hours, in 1950 still 2000 but at present only 1600 on average. Simultaneously, life expectancy rose from 45 to almost 80 years due to the advances in medical sciences, nutrition and sanitary installations and the GDP increased from some 2,000 PPPUS$ well beyond 20,000 PPPUS$ due to higher productivity [27] . Here, US$ as currency unit corrected for purchasing power parity are used throughout. Higher life quality, therefore, was not only achieved through longer lives and higher consumption but also by significantly more leisure time. It has even been argued repeatedly on philosophical, sociological and economical levels that time for leisure is the ultimate source of life quality[57] .

Then, some elegant mathematical derivation in [25] lead to the traditional form of the *Life Quality Index (LQI)*

$$L \propto \frac{g^q}{q} e \tag{12}$$

where, for later convenience, $q = \frac{w}{1-w}$. The fraction of time w of e necessary for paid work varies between 0.12 and 0.25 (see [34] for estimates of w for different countries). Nathwani et al. (1997) assume that $L = f(g)h(t)$ with $t = (1 - w)e$ and where t is the fraction of life devoted to leisure and we the fraction of life devoted to paid work. Thus, the LQI is a product of a function $f(g)$ measuring life quality and a function $h(t)$ measuring the duration of enjoyment of life. Defining relative changes in the LQI by $\frac{dL}{L} = \frac{g}{f(g)} \frac{df(g)}{dg} \frac{dg}{g} + \frac{t}{h(t)} \frac{dh(t)}{dt} \frac{dt}{t} = k_g \frac{dg}{g} + k_t \frac{dt}{t}$ and setting $k_g / k_t = const.$ according to the universality requirement, one finds two differential equations $k_g \equiv \frac{g}{f(g)} \frac{df(g)}{dg} = c_1$ and $k_t \equiv \frac{t}{h(t)} \frac{dh(t)}{dt} = c_2$ with solutions $f(g) = g^{c_1}$ and $h(t) = t^{c_2} = ((1-w)e)^{c_2}$. Assume further that $g \propto cw$ where c is the productivity of work. *"Presumably, people on the average work just enough so that the marginal value of wealth produced, or income earned, is equal to the marginal value of the time they lose when at work* " [25] . Consequently, people who work, possibly together with their families, optimize work and leisure time, i.e. their LQI. From $\frac{dL}{dw} = 0$ one determines $c_1 = c_2 \frac{w}{1-w}$ which together with $c_1 + c_2 = 1$ results in $L = g^w e^{1-w} (1-w)^{1-w} \approx g^w e^{1-w}$.

Additionally, we take the $1/(1 - w)$-th root and divide g^q by $q = \frac{w}{1-w}$ which gives eq. (12).

Dividing g^q by q removes a minor inconsisteney of the original form because persons with the same g and e but larger w would have higher life quality.

Using eq. (11) yields a general acceptance criterion for investments into projects for risk reduction:

$$\frac{dg}{g} + \frac{1}{q} \frac{de}{e} \geq 0 \tag{13}$$

The equality in (13) gives an indication of what is *necessary* and *affordable* to a society for life saving undertakings, projects having "<" are not admissible. The latter projects would, in fact, be life-consuming and, thus, be in conflict with the constitutional right to live. Whenever a given incremental increase in life expectancy by some life saving operation (positive de) is associated with larger than optimal incremental cost (negative dg) one should invest into alternatives of life saving. If a given positive de can be achieved with less than required by eq. (13) it should be done, of course. Eq. (13) is easy to interpret. For example, a 1% increase in life expectancy requires yearly investments of about 5% of g for $q = 0.2$. From a practical point of view it is important that all quantities on the right-hand side of eq. (13) are easily available and can be updated any time. The democratic equality principle dictates that average values for *g, e* and *w* have to be taken. Any deviations from average values for any specific group of people need to be justified carefully if eq. (13) is applied to projects with involuntary risks.

There is a certain dilemma arising from the actual unequal distribution of wealth and life expectancy in a society. A certain group in a society may benefit from safety interventions more than another. Then, it should be fair that the "gainers" compensate the "losers" so that their LQI is at least maintained. For example, in projects where certain groups of people must take higher risks, voluntarily or involuntarily, it should be fair to provide compensation by higher incomes or more leisure time. One even may follow a requirement in [24] which states that the "gainers" should still have some left over. Similar "solidarity" principles should also apply if only a certain group in society is exposed to some hazards. Much further discussion is provided in [21] and [25] .

Life quality clearly has more dimensions than consumption, life expectancy and leisure time. Values such as personal well-being, good family relationships, a healthy ecological environment, cultural heritage and many other values cannot be measured by the life quality index. However, we only intend to derive a criterion helping to balance conflicting aims in a rational manner.

Practical application of eq. (13) requires estimation of dg/g and de/e. In general, the cost involved in some life saving operation can be determined easily. The estimation of the effect of a life saving operation is more difficult. We start by estimating the cost of averting a fatality in terms of the gain in life expectancy Δe. The cost of the safety measure is expressed as a reduction Δg of the GDP. This ***l**ife **s**aving **c**ost* (*LSC*)or ***i**mplied **c**ost of **a**verting a **f**atality* (*ICAF*) can be obtained from the equality of eq. (13) after separation and integration from g to $g + \Delta g$ and e to $e + \Delta e$ i.e. the cost $\Delta C = -\Delta g$ per year to extend a person's life by Δe is:

$$\Delta C = -\Delta g = g\left[1 - \left(1 + \frac{\Delta e}{e}\right)^{-\frac{1}{q}}\right]$$

Because ΔC is a yearly cost and the (undiscounted) *LSC* has to be spent for safety related investments into technical projects at the decision point $t = 0$, one should multiply by $e_r = \Delta e$ and

$$LSC(e_r) = g\left[1 - \left(1 + \frac{e_r}{e}\right)^{-\frac{1}{q}}\right]e_r \tag{14}$$

follows. The societal equality principle prohibits to differentiate with respect to special ages within a group. The conditional (remaining) life expectancy given that the person has survived up to age a is:

$$e(a) = \int_a^{a_u} \frac{\ell(t)}{\ell(a)}\,dt = \frac{1}{\ell(a)}\int_a^{a_u} exp\left[-\int_0^t \mu(\tau)d\tau\right]dt \tag{15}$$

Therefore, averaging the remaining life expectancy over the age distribution leads to the ***s**ocietal* ***l**ife* ***s**aving* ***c**ost* (*SLSC*)

$$SLSC = \int_0^{a_u} LSC\,(e(a))\,h\,(a,n)\,da \tag{16}$$

where $h(a, n)$ is the density of the age distribution of the population with n its population growth rate. The density of the age distribution can be obtained from life tables. For a stable population it is given by:

$$h(a,n) = \frac{\exp\,[-na]\,\ell(a)}{\int_0^{a_u} \exp\,[-na]\,\ell(a)\,da} \tag{17}$$

A stationary population is obtained for $n = 0$ so that $h(a) = \ell(a)/e$. In countries with a fully developed social system *SLCS* is approximately the amount to support the (not working) relatives of the victims of an event by the social system, mostly by redistribution. If no social system is present, it is useful to think of the amount an insurance should cover after an event. For example, if $GDP \approx$ 25,000 PPP US$ and thus, $g \approx$ 15,000 PPP US$, $e \approx 77$ years and $w \approx 0.14$ one calculates $SLSC \approx$ 600,000 PPP US$.

The direct quantification of de/e is difficult but there is a good approximation if life saving operations result in certain forms of small changes of age-dependent mortality rates. We start with the assumption that crude mortality is changed by dm. For a (small) uniform proportional change, i.e. $dm = \pi m$ or $\pi = dm\text{-}m$ in age dependent mortality $\mu(a)$ i.e. $\mu_\pi\,(a) = \mu(a)\,(1+\pi)$ the change in de/e is [20]

$$\frac{de}{e} \approx \frac{\frac{d}{d\pi}\int_0^{a_u} \exp\left[-\int_0^{a} (\mu(\tau)(1+\pi))d\tau\right] da\Big|_{\pi=0}}{\int_0^{a_u} \ell(a)\,da}\pi = \frac{\frac{d}{d\pi}\int_0^{a_u} \ell(a)^{1+\pi}\,da\Big|_{\pi=0}}{\int_0^{a_u} \ell(a)da}$$

$$= -\frac{\int_0^{a_u} ln(\ell(a))\,\ell(a)\,da}{\int_0^{a_u} \ell(a)\,da}\pi = -c_\pi \frac{dm}{m} = -C_\pi dm \tag{18}$$

where $c_\pi \approx 0.15$ (developed countries) to more than 0.5 (some developing countries) depending on the age structure and life expectancy of the group and therefore $C_\pi \approx 0.15$ (see [34] for more details). This scheme places the majority of the profit of a mortality reduction on older people and, therefore, is considered as not compatible with the equality principle in modern societies. However, if mortality is delayed as for some air pollution substances it may very well be used as an approximation. Eq. (18) and the like are valid for positive as well as negative dm. Alternatively, one can assume that a (small) change $dm = \Delta$ in crude mortality distributes equally as a constant at all ages. Then, $\mu(a)$ changes into $\mu_\Delta(a) = \mu(a) + \Delta$ and one has

$$\frac{de}{e} \approx \frac{\frac{d}{d\Delta}\int_0^{a_u} \exp\left[-\int_0^a \left(\mu(\tau)+\Delta\right)d\tau\right] da_{\,|\Delta=0}}{\int_0^{a_u} \ell(a)\,da}\Delta$$

$$= -\frac{\int_0^{a_u} a\ell(a)\,da}{\int_0^{a_u} \ell(a)\,da}\Delta = -C_\Delta dm = -c_\Delta \frac{dm}{m} \tag{19}$$

with $c_\Delta = C_\Delta m$. In this case the constants c_Δ are around 0.35 ($C_\Delta \approx 35$) for developed countries. For a given *dm* the changes in $\frac{de}{e}$ become roughly twice as large. This must be expected because a constant change of $\mu(a)$ in young ages has substantially more effect on life expectancy than in older ages. For technical applications, e.g. in structural reliability, industrial hazard protection, flood protection, earthquake-resistant design, etc., this is probably the most realistic and fair regime. The influence of the particular age distribution can be significant.
Other mortality regimes can be thought of. For example, one can also consider age dependent mortality regimes if a change in mortality only affects those older than 60 years or any other age group as might be relevant in health-related public investments. The selection of the appropriate mortality regime turns out to be rather important in applications.
Using eq. (18) or (19) in eq. (13) leads to the yearly cost of a risk-reducing intervention

$$dC_Y = -dg = g\frac{1}{q}\frac{c_x}{m}dm = G_x dm \tag{20}$$

The index "x" stands for either "π", "Δ" or any other mortality regime. With m = 0.01 and c_π = 0.15 and $c_\Delta \approx 0.35$ or $c_\pi = 0.5$ but otherwise the same data as before one calculates $G_\Delta \approx$ 1,300,000 PPPUS\$ or $G_\Delta \approx$ 3,200,000 PPPUS\$. The (yearly) quantity (20) is denoted as " *willingness-to-pay* " in health-oriented economical studies.
So far, we concentrated on life saving cost and the willingness-to-pay for averting fatalities and neglected the cost implied by injuries. This appears justified as the latter are relatively small. For instance, the study in [14] suggests that, for the United States, the cost of injury can be taken as 1000 US\$ / person and 10000 US\$ / person for minor and serious injury, respectively. These numbers are by orders of magnitude smaller than those determined on the basis of the LQI and by other approaches (see next section and table 3).

3 Further socio-economic considerations

Health-related economics has developed similar concepts. Denote by $c(\tau) > 0$ the consumption rate at age τ and by $u(c(\tau))$ the utility derived from consumption. Individuals tend to undervalue a prospect of future consumption as compared to that of present consumption. This is taken into account by discounting. The life time utility for a person at age a until she / he attains age $t > a$ then is

$$U(a,t)=\int_a^t u\left[c(\tau)\right]\exp\left[-\int_a^\tau \rho(\theta)d\theta\right]d\tau=\int_a^t u[c(\tau)]\exp\left[-\rho(\tau-a)\right]d\tau \qquad (21)$$

for constant $\rho(\theta) = \rho$ Note that discounting is with respect to utility. It is assumed that consumption is not delayed, i.e. incomes are not transformed into bequests. ρ should be conceptually distinguished from a financial interest rate and is referred to as rate of time preference of consumption. A rate $\rho > 0$ has been interpreted as the effect of human impatience, myopia, egoism, lack of telescopic faculty, etc. It is partially justified because there is uncertainty about one's future. The economics literature also states that if no such "discounting" is applied more emphasis on the well being of future generations is placed rather than improving welfare of those alive at present, assuming economic growth. Exponential population growth with rate n can be considered by replacing ρ by $\rho - n$ taking into account that families are by a factor *exp[nt]* larger at a later time $t > 0$. Exponential population growth can easily be verified from the data collected in [27] . The correction $\rho > n$ appears always necessary, simply because future generations are expected to be larger recalling that utility of consumption is always referred to a single person. As mentioned, future generations are also wealthier due to economic growth. Therefore, one should add the exponential growth rate ζ or, alternatively, one thinks of ρ to include economic growth by ζ. Exponential growth can again be verified from the data in [27] as a good approximation. In contrast to [35] the economic growth rate is taken into account explicitely. A rate $\rho + \zeta > n$ is necessary for eq. (21) to converge if future generations are included, i.e. if the utility integral must be extended to $t \to \infty$. ρ is reported to be between 1 and 4% for health related investments, with tendency to lower values [51]. Empirical estimates reflecting pure consumption behavior vary considerably but are in part significantly larger [19]. The expected remaining present value life time utility at age a (conditional on having survived until ρ) then is (see [3] [41] [38] [11])

$$L(a)=E[U(a)]=\int_a^{a_u}\frac{f(t)}{\ell(a)}U(a,t)dt$$

$$=\frac{1}{\ell(a)}\int_a^{a_u}u[c(t)]\exp\left[-(\rho+\zeta-n)(t-a)\right]\ell(t)dt=u[c]e_d(a,\zeta,\rho,n) \qquad (22)$$

where $f(t)dt=\left(\mu(\tau)\exp\left[-\int_0^t\mu(\tau)d\tau\right]\right)dt$ is the probability of dying between age t and $t + dt$ computed from life tables. The expression in the second line is obtained upon integration by parts. Also, a constant consumption rate c independent of t has been introduced which can be shown to be optimal under perfect market conditions [41] . Note that $L(a)$ is finite throughout due to $a_u < \infty$. The "discounted" life expectancy $e_d(a,\zeta,\rho,n)$ at age a can be computed from

$$e_d(a,\zeta,\rho,n)=\frac{\exp\left((\rho+\zeta-n)a\right)}{\ell(a)}\int_a^{a_u}\exp\left[-\int_0^t\left(\mu(\tau)+(\rho+\zeta-n)\right)d\tau\right]dt \qquad (23)$$

"Discounting" affects $e_d\left(a,\zeta,\rho,n\right)$ primarily when $\mu\left(\tau\right)$ is small (i.e. at young age) while it has little effect for larger $\mu\left(\tau\right)$ at higher ages. It is important to recognize that "discounting" by ρ is initially with respect to $u[c(\tau)]$ but is formally included in the life expectancy term. Clearly, there is $e_d\left(0,0,\rho,0\right)\leq e$ for $\rho > 0$. For simplification of presentation it is also assumed that the quantity $\mu\left(\tau\right)$ and therefore also $\ell\left(\tau\right)$ do not change over time, for example due to further progress in medical sciences.

For $u[c]$ we select a power function

$$u\left[c\right]=\frac{c^q-1}{q} \tag{24}$$

with $0\leq q\leq 1$ implying constant relative risk aversion according (CRRA) to Arrow-Pratt. The form of eq. (24) reflects the reasonable assumption that marginal utility $\frac{du[c]}{dc}=c^{q-1}$ decays with consumption c. $u[c]$ is a concave function (to the below) since $\frac{du[c]}{dc}>0$ for $q\geq 0$ and $\frac{d^2u[c]}{dc^2}<0$ for $q < 1$. The value of q is further discussed below. For simplicity, we take $c = g >> 1$. Shepard / Zeckhauser [41] now define the "value of a statistical life" at age a by converting eq. (22) into monetary units in dividing it by the marginal utility $\frac{du(c(t))}{dc(t)}=u'[c(t)]$:

$$\begin{aligned} VSL(a) &= \int_a^{a_u}\frac{u[c(t)]}{u'[c(t)]}\exp[-(\rho+\zeta-n)(t-a)t]\frac{\ell(t)}{\ell(a)}dt \\ &= \frac{u[c]}{u'[c]}\frac{1}{\ell(a)}\int_a^{a_u}\exp\left[-(\rho+\zeta-n)(t-a)\right]\ell(t)\,dt \\ &= \frac{g}{q}\frac{1}{\ell(a)}\int_a^{a_u}\exp\left[-(\rho+\zeta-n)(t-a)\right]\ell(t)\,dt \quad = \quad \frac{g}{q}e_d(a,\zeta,\rho,n) \end{aligned} \tag{25}$$

because $\frac{u[c(t)]}{u'[c(t)]}=\frac{g}{q}$. It is seen that $VSL(a)$ decays with age as $e_d\left(a,\zeta,\rho,n\right)$. The "willingness-to-pay" has been defined as

$$WTP\ (a) = VLS\ (a)\ dm \tag{26}$$

Obviously, the mortality regime in eq. (19) is assumed but a generalization to other mortality regimes should be possible. In analogy to Pandey / Nathwani [30] , and here we differ from the related economics literature, these quantities are averaged over the age distribution $h(a, n)$ in a stable population in order to take proper account of the composition of the population exposed to natural or man-made hazards. This defines the "societal value of a statistical life"

$$\overline{SVSL}=\frac{g}{q}\bar{E}(\zeta,\rho,n) \tag{27}$$

with the age-averaged, discounted life expectancy

$$\overline{E}(\zeta,\rho,n)=\int_0^{a_u} e_d(a,\zeta,\rho,n)h(a,n)da \tag{28}$$

and the “societal willingness-to-pay” as:

$$\overline{SWTP}=\overline{SVSL}\, dm \tag{29}$$

For $\rho = 0$ the averaged “discounted” life expectancy $\overline{E}(\rho,n)$ is a quantity which is about 60% of e and considerably less than that for larger ρ. It is easily shown that the elasticity of $\overline{SVSL}$ with respect to income is one.

In this purely economic consideration it appears appropriate to define the equivalent to the *SLSC* as the undiscounted average lost earnings in case of death, i.e. the so-called “human capital”, averaged over the age distribution eq. (15):

$$SHC = g\int_0^{a_u} e(a)\, h\,(a,n)\, da \tag{30}$$

One can show that *SHC* is slightly larger than *SLSC*. Table 1 shows the $\overline{SVSL}$ for some selected countries as a function of ρ indicating the importance of a realistic assessment of ρ.

Table 1: $\overline{SVSL}$ 10^6 in PPP US\$ for some countries for various $\rho + \zeta$ (from recent complete life tables from national statistical offices)

		France	**Germany**	**Japan**	**Netherlands**	**USA**
e		78	78	80	78	77
n		0.37%	0.27%	0.17%	0.55%	0.90%
g		14660	14460	15960	15470	22030
q		0.135	0.132	0.153	0.111	0.174
	0%	5.22	5.01	4.70	7.19	7.44
	1%	3.93	3.80	3.56	5.37	5.46
$\rho + \zeta$	2%	3.07	2.99	2.80	4.17	4.17
	3%	2.48	2.43	2.27	3.34	3.30
	4%	2.05	2.01	1.89	2.75	2.69

Inspection of eq. (22) with (24) and integrating over the age distribution $h\,(a, n)$, however, reveals exactly eq. (12) with e replaced by $\overline{E}(\zeta,\rho,n)$. It has been called *Societal Life Quality Index* (*SLQI*) by Pandey / Nathwani [30].

$$L_{\overline{E}}=\frac{g^q}{q}\int_0^{a^u} e_d(a,\zeta,\rho,n)\, h\,(a,n)\, da=\frac{g^q}{q}\,\overline{E}(\zeta,\rho,n) \tag{31}$$

It is to be emphasized that the SLQI, like the original LQI, is not a monetary quantity and has dimension "(US$)^q (years)". It should interpreted as a utility function. If divided by the marginal utility $u'(c)$ it coincides with eq. (27).

The numerical value $q = \frac{w}{1-w}$ may be derived from the work-leisure optimization principle underlying eq. (12). Using this principle one obtains $q \approx 0.2$ from estimates of w in [34] and elsewhere which agrees well with estimates used in [41] [11] (see also table 3 below). This magnitude of q has also been verified empirically (see, for example, [19]). It can be seen from table 3 below that societies with larger g generally work less whereas people in countries with smaller g work more in order to increase utility of consumption. However, in some countries more preferences are given to large earnings and thus large consumption whereas other societies prefer larger leisure time. Obviously, other secondary factors also affect the value of q. This is also supported by recent labor statistics [28] [29] [12]. It is noteworthy that the power function form of eq. (24) is also the result of the derivations for eq. (12).

The reasoning in eqs. (9) to (13) offers the possibility to arrive at a slightly different criterion for the willingness-to-pay. Define a new coefficient relating changes in mortality to changes in averaged "discounted" life expectancies for given mortality regimes, similar to eq. (18) or (19):

$$\frac{d\bar{E}}{\bar{E}} \approx \frac{\frac{d}{dx}\bar{E}(x)\big|_{x=0}\, x}{\bar{E}} = -C_{x\bar{E}}(\zeta,\rho,n)\,dm = -\frac{c_{x\bar{E}}(\zeta,\rho,n)}{m}\,dm \tag{32}$$

The formulae are lengthy and are not given here. The coefficients $C_{x\bar{E}}(\zeta,\rho,n)$ for averaged "discounted" life expectancies turn out to be somewhat larger than those computed with "undiscounted" and not averaged life expectancies. The coefficients $C_{x\bar{E}}(\zeta,\rho,n)$ are all decreasing while ρ increases, but at different speed. Table 2 shows the coefficients $C_{x\bar{E}}(\zeta,\rho,n)$ for some countries. The population growth rates n have been taken into account according to [10].

Table 2 is interesting because it shows the significant but rather complex influence of demographic factors. For information the mean age $\bar{e}$ of the population is also given from which the residual mean life expectancy can be calculated. Comparing the results with the results in table 1 indicates that the influence of $\rho + \zeta$ is significantly larger in table 1 than in table 2. The influence of the mortality reduction scheme is remarkable. To illustrate this further consider the additive mortality reduction scheme eq. (19) but now the mortality reduction affects only those under 18 years, between 18 and 60 and above 60, respectively. Such strategies might be suitable for certain risk reduction interventions in pollution control of water or atmosphere. For example, for the USA one determines coefficients $C_{\Delta\bar{E}}$ of 9.5, 8.6 and 0.7, respectively. These values, of course, add up to the value for a constant mortality change at all ages.

Table 2: Dependence of $C_{\pi\bar{E}}$ and $C_{\Delta\bar{E}}$ on rate $\rho + \zeta$

		France	Germany	Japan	Netherlands	USA
$e(\bar{e})$		78 (37.9)	78 (38.3)	80 (39.9)	78 (36.6)	77 (34.0)
n		0.37%	0.27%	0.17%	0.55%	0.90%
	0%	26, 30	22, 29	26, 29	27, 30	33, 32
	1%	22, 26	18, 26	22, 25	22, 26	27, 28
$\rho + \zeta$	2%	19, 22	16, 21	19, 21	17, 23	26, 24
	3%	16, 19	13, 19	16, 19	16, 19	19, 21
	4%	14, 17	12, 16	14, 16	14, 17	16, 18

Application of the reasoning in Nathwani et al. [25] leads to the same form as in eq. (13) with e replaced by $\bar{\mathbf{E}}$:

$$\frac{dg}{g} + \frac{1}{q}\frac{d\bar{E}}{\bar{E}} \approx \frac{dg}{g} - \frac{1}{q} C_{x\bar{E}}(\zeta,\rho,n)dm \approx \frac{dg}{g} - \frac{1}{q}\frac{c_{x\bar{E}}(\zeta,\rho,n)}{m}\, dm \geq 0 \tag{33}$$

Rearrangement then produces a formula also expressing the "willingness-to-pay"

$$dC_Y = g\frac{1}{q} C_{x\bar{E}}(\zeta,\rho,n)dm = g\frac{1}{q}\frac{c_{x\bar{E}}(\zeta,\rho,n)}{m}\, dm = G_{x\bar{E}}(\zeta,\rho,n)dm \tag{34}$$

$G_{x\bar{E}}(\zeta,\rho,n)$ in eq. (34) contains implicitly or explicitly crude mortality which in this context can also be called background mortality, i.e. the specific mortality in a group due to other causes of death including those of natural death. It is remarkable that in both cases eq. (27) and (34) the societal willingness-to-pay is proportional to the amount g of GDP available for risk reduction and some demographic constant (either $\bar{E}(\zeta,\rho,n)$ or $G_{x\bar{E}}(\zeta,\rho,n)$) and inversely proportional to the risk aversion parameter q.

For the same data as used for *SLSC* above and $m \approx 0.01$, $n \approx 0.003$, $\delta \approx 0.019$, $\rho \approx 0.006$, a European life table and, therefore, $C_{\pi\bar{E}}(\zeta,\rho,n) \approx 16$, the constant $G_{\pi\bar{E}}(\zeta,\rho,n)$ is $1.7 \cdot 10^6$ PPP US\$. If one adopts the mortality regime in eq. (19) we have $C_{\Delta\bar{E}}(\zeta,\rho,n) \approx 19$ and $G_{\Delta\bar{E}}(\zeta,\rho,n) \approx 2.1 \cdot 10^6$PPP US\$. These values are to be compared with $\overline{SVSL} = 2.5 \cdot 10^6$ PPP US\$. Neglecting discounting altogether gives, for example, $C_{\Delta\bar{E}}(0,0,0) \approx 26$ and, therefore, $G_{\Delta\bar{E}}(0,0,0) \approx 2.9 \cdot 10^6$ PPP US\$. No age averaging and no discounting results, for example, in $G_{\Delta} \approx 4.3 \cdot 10^6$ PPP US\$ and, therefore, is at the upper end of the estimates.

Both lines of thought, the economical and the LQI approach, have a good conceptual and theoretical basis. They complement each other. In particular, the derivations for eq. (12) justify the power function form in eq. (24) and lets eq. (31) be interpreted as an expected remaining present value life time utility for all those alive at $t = 0$. Neither criterion (34) nor (29) depend on any benefit other than risk reduction or life extension. In most applications clear support for decisions can be reached by using either of the approaches, even the one without discounting and age averaging. But it is believed that age averaging is generally necessary for the technical applications we have in mind because the risk reduction intervention is to be executed at $t = 0$ for all living now and for all living in the future or, more precisely, approximately within the next 100 years. The concept of discounting future utilities by $(\rho + \zeta - n)$ may be debatable as the subjective time preference rate ρ is concerned but not with respect to the population and economic growth. The *SLQI*-based approachc i.e. $G_{x\overline{E}}(\zeta, \rho, n)$, explicitly combines three important human concerns, that is high life expectancy, high consumption and an optimized time available for the development of one's personality off the time for paid work. Criterion (34), having in mind its derivation, also tells us that larger expenses for risk reduction are inefficient and smaller expenses are not admissible in view of the constitutional right for life. In particular, criterion (34) is affordable from a societal point of view. Eq. (34) deserves the name "societal willingness-to-pay" even in a more direct sense than eq. (29) as it is the result of some optimization of time of work to raise the income and leisure time given a certain productivity of the economy. Insofar the *SLQI*-concept appears to be somewhat richer and more suitable for our purposes than the purely economic approach leading to eq. (27) and (29). Finally, the lack of theory in the economic approach does not allow to consider different mortality regimes for the time being. The "willingness-to-pay" according to eq. (34) should replace the one in eq. (20) except for cases in which the more general and probably more realistic concept leading to eq. (34) does not apply.

Are similar adjustments with respect to discounting also necessary for the *SLSC* or the *SHC*? The author is inclined to negate it because the compensation cost calculated approximately by the *SLSC* or the *SHC* in eq. (16) and eq. (30), respectively, become real in an adverse event and have to be carried by the social system or insurance or both. Also, double discounting must be avoided if *SLSC* or *SHC* are used in equations of the type (3).

Finally, our considerations are limited to non-catastrophic adverse events, i.e. events which do not substantially change the demographic structure of the group under discussion and which do affect the regional or even national economy only very little.

4 Application to technical facilities

It can reasonably be assumed that the life risk in and from technical facilities is uniformly distributed over the age and sex of those affected. Also, it is assumed that everybody uses such facilities and, therefore, is exposed to possible fatal accidents. The total cost of a safety related regulation per member of the group and year is $dg = -dC_Y(p) = -\frac{1}{N}\sum_{i=1}^{8} dC_{Y,i}(p)$ where s is the total number of objects under discussion, each with incremental cost $dC_{Y,i}$ and N is the group size. For simplicity, the design parameter is temporarily assumed to be a scalar. Inserting into eq. (33) gives:

$$\frac{-dC_Y(p)}{g}+\frac{1}{q}\left(-C_{x\overline{E}}(\zeta,\rho,n)dm\right)\geq 0$$

Let dm be proportional to the mean failure rate $dh(p)$, i.e. it is assumed that the process of failures and renewals is already in a stationary state that is for $t\to\infty$ (see eq. (5)).

Rearrangement yields

$$\frac{dC_Y(p)}{dh(p)}\geq -kC_{x\overline{E}}(\zeta,\rho,n)g\frac{1}{q}=-kG_{x\overline{E}}(\zeta,\rho,n) \quad (35)$$

where

$$dm = kdh(p), 0 < k \leq 1 \quad (36)$$

the proportionality constant k relating the changes in mortality to changes in the failure rate. Note that for any reasonable risk reducing intervention there is necessarily $dh(p) < 0$.
The criterion eq. (35) is derived for safety-related regulations for a larger group in a society or the entire society. Can it also be applied to individual technical projects? $G_{x\overline{E}}(\zeta,\rho,n)$ as well as *SLSC* were related to one anonymous person. For a specific project it makes sense to apply criterion (35) to the whole group exposed. Therefore, the "life saving cost" of a technical project with N_{PE} potentially endangered persons is:

$$H_F = SLSC\; kN_{PE} \quad (37)$$

The monetary losses in case of failure are decomposed into $H = H_M + H_F$ in formulations of the type eq. (3) with H_M the losses not related to human life and limb.
Criterion (35) changes accordingly into:

$$\frac{dC_Y(p)}{dh(p)}\geq -G_{x\overline{E}}(\zeta,\rho,n)\,kN_{PE} \quad (38)$$

All quantities in eq. (38) are related to one year. For a particular technical project all design and construction cost, denoted by $dC(p)$ must be raised at the decision point $t = 0$. The yearly cost must be replaced by the erection cost $dC(p)$ at $t = 0$ on the left hand side of eq. (38) and discounting is necessary. The method of discounting is the same as for discharging an annuity. If the public is involved $dC_Y(p)$ may be interpreted as cost of societal financing of $dC(p)$. The (real) interest rate to be used must then be a societal interest rate to be discussed below. Otherwise the interest rate is the market rate. g in $G_{x\overline{E}}(\zeta,\rho,n)$ also grows in the long run approximately exponentially with rate $\delta=\zeta-n$ the rate of economic growth in a country (see [27] for an empirical verification). It can be taken into account by discounting. The acceptability criterion for individual technical projects then is (discount factor for discounted erection cost moved to the right hand side):

$$\frac{dC(p)}{dh(p)} \geq -\frac{\exp[\gamma t_s]-1}{\gamma \exp[\gamma t_s]} G_{x\overline{E}}(\zeta,\rho,n) \frac{\delta \exp[\delta t_s]}{\exp[\delta t_s]-1} kN_{PE} \underset{t_s \to \infty}{\longrightarrow} -G_{x\overline{E}}(\zeta,\rho,n) kN_{PE} \frac{\delta}{\gamma} \quad (39)$$

where t_s is service time. For $\delta \to 0$ as well as $\gamma \to 0$ we have the interesting limiting result for arbitrary t_s:

$$\frac{dC(p)}{dh(p)} \geq \underset{\delta \to 0, \gamma \to 0}{\longrightarrow} -G_{x\overline{E}}(\zeta,\rho,n) kN_{PE} \quad (40)$$

The same derivations apply to the purely economic concept with $G_{x\overline{E}}(\zeta,\rho,n)$ replaced by $\overline{SVSL}$.

N_{PE}. as well as k must be estimated taking account of the number of persons endangered by the event, the cause of failure, the severity and suddenness of failure, possibly availability and functionality of rescue systems, etc. The constant k may be interpreted as a person's probability of actually being killed in case of failure. It can vary between less than 1 / 10000 and 1. In practice the estimation of N_{PE}. and k is the subject of risk analysis or better failure consequence analysis. In general, it can only be made for specific projects. It should also be noted that the probability k and the particular mortality regime can depend on each other. Further discussions of the methodology to determine the parameters N_{PE} and k as well as the particular mortality regime are beyond the scope of this paper.

5 Optimization for technical components

For the special task in eq. (3) we have

$$\text{Maximize: } Z(\mathbf{p}) = \frac{b(\mathbf{p})}{\gamma} - C(\mathbf{p}) - (C(\mathbf{p}) + H_M + H_F)\frac{\lambda P_f(\mathbf{p})}{\gamma}$$

$$\text{Subject to: } f_k(\mathbf{p}) \le 0, k = 1,...,q$$

$$\nabla_p C(\mathbf{p}) + G_{x\overline{E}}(\zeta,\rho,n)kN_F \frac{\delta}{\gamma}\nabla_p(\lambda P_f(\mathbf{p})) \ge 0 \tag{41}$$

where the first condition represent some restrictions on the vector **p** of optimization variables_ Of course, $\nabla_p C(\mathbf{p})$ is assumed to increase in all component of **p** and $\nabla_p(\lambda P_f(\mathbf{p}))$ to decrease.

The first condition represents limits on the parameter vector **p**. The second condition represents the LQI -acceptability criterion written out for vectorial parameter **p** and an infinite time horizon. As before the failure consequences are decomposed into direct cost H_M (including indirect failure cost such as loss of business, service, etc.) and life saving cost H_F defined in eq. (37). Technical details for the solution of the problem in eq. (41) are summarized in [43].
The formulation eq. (41) includes the SLQI-criterion eq. (39). Assume that the conditions $f_k(\mathbf{p}) \le 0$ are not active in the solution point and $b = b(\mathbf{p})$. At the optimum there must be $\nabla_p Z(\mathbf{p}) = 0$ i.e. for $\mathbf{p} = \mathbf{p}^*$

$$\nabla_p C(\mathbf{p})(1 + \lambda P_f(\mathbf{p})) + \left[\frac{(C(\mathbf{p}) + H_M + H_F)}{\gamma}\nabla_p(\lambda P_f(\mathbf{p}))\right] = 0 \tag{42}$$

which is to be compared with the equality of eq. (39) written out for vectorial parameter **p**:

$$\nabla_p C(\mathbf{p}) + G_{x\overline{E}}(\zeta,\rho,n)kN_{PE}\frac{\delta}{\gamma}\nabla_p(\lambda P_f(\mathbf{p})) = 0 \tag{43}$$

By neglecting the small quantity $\lambda P_f(\mathbf{p})$ in the first term of eq. (42) we see that if there is $(C(\mathbf{p}) + H_M + SLSC\ kN_{PE})/\gamma \ge G_{x\overline{E}}(\zeta,\rho,n)kN_{PE}\frac{\delta}{\gamma}$ the optimal solution for eq. (3) will automatically fulfill the SLQI-criterion eq. (39). It can be shown that this is frequently the case under conditions of interest. It will almost always be true if $\frac{\delta}{\gamma} \approx 1$ as will be shown in the next section, and if the same γ is used in eq. (42) and (43). Therefore, optimal structures are almost always safer than the SLQI-criterion would require. Eq. (43) also provides an alternative interpretation of the SLQI-criterion (39) because we can recover the following optimization task:

$$\text{Minimize: } Z'(\mathbf{p}) = C(\mathbf{p}) + G_{x\overline{E}}(\zeta,\rho,n)kN_{PE}\frac{\delta}{\gamma}(\lambda P_f(\mathbf{p})) \tag{44}$$

Eq. (43) is seen to be the optimality condition $\nabla_p Z'(\mathbf{p}) = 0$ of the (unconstrained) optimization problem eq. (44). Eq. (44) allows solving for vectorial parameter **p**. A solution to eq.(43) or (44) can always be found because $\nabla_p C(\mathbf{p})$ usually grows approximately linearly in **p** whereas $\nabla_p (\lambda P_f(\mathbf{p}))$ decays exponentially. Some further implications of the foregoing results cannot be discussed herein.

6 Societal discount rates

The cost for saving life years in eq. (37) enters into the objective function (3) and with it the question of discounting those cost arises. Also, a discount rate is present in eq. (39). At first sight discounting of human lives is not in agreement with our moral value system. However, a number of studies summarized in [31] and [22] express a rather clear opinion based on ethical and economical arguments. The cost for saving life years must be discounted at the same rate as other investments. Otherwise serious inconsistencies cannot be avoided.

What should then the societal interest rate be? In view of the time horizon of some 20 to more than 100 years (i.e. several generations) it should be a long-term average. It should be net of inflation and taxes. For $\gamma = 0.075$, 1$ benefit (or loss) in 100 years is presently worth less than 0.1 cent, which appears unacceptable if human lives (in present and future generations) are concerned. But $\gamma = 0.015$ gives 0.23 $, which lets us feel a little more comfortable, yet still unsatisfied. The long time horizon generally suggests to prefer small rates. Weinstein / Stason [54] require that interest rates for life saving investments should be the same as for other cost and thus equal to the real market interest rate, simply for consistency reasons. This appears to be an extreme point of view. The other extreme of not discounting intergenerationally at all is expressed, for example, in [8] and [40] , based primarily on ethical grounds in the context of CO_2 -induced global warming, nuclear waste disposals, depletion of natural resources, etc. In this case the rationale of our basic optimization model eq. (3) together with eq. (39) and part of the considerations in chapter 2 break down. Presumably, there is something in between which can be founded rationally.

There have been ongoing but somewhat inconclusive discussions when discounting public investments into health care (see, for example, [51]). Recently, further discussions have been taken place in the context of sustainable development, long term public investments in general and intergenerational justice -aspects which appear very relevant in our context. Discounting for sustainability should at least be consistent with discounting for risk reduction investments. Due to the requirement $\beta > \gamma_{max}$ stated just below eq. (7), the interest rate is strongly related to the benefit a society earns from its various activities, i.e. its real economic growth. The United Nations Human Development Report 2001 [48] gives values between 1.2 and 1.9 % for industrialized countries during 1975-1998. If one considers the last 120 years and the data in [27] for some selected countries one determines a growth rate $\delta = \frac{\ln(g_{1992}/g_{1870})}{1992-1870}100$ of about 1.8% which is also the growth rate for Western Europe, the so-called Western Offshots, USA, Canada and Australia, and Japan. For Southern Europe, Latin America and Asia one finds from the same data similar growth rates $\bar{\delta} \approx 1.7\%$, for Eastern Europe still $\bar{\delta} \approx 1.4\%$ but for Africa only $\bar{\delta} \approx 0.9\%$.

Some more insight can be gained from modern economic growth theory but also sustainability financing. Nordhaus [26] and others (see [47] for an overview but also the other papers in Energy Policy, 23, 3/4, 1995) follow the classical Ramseyan approach (see [37] [42] and [5]) for optimal stable economic growth in perfect markets

$$\gamma = \rho + \varepsilon\delta > 0 \tag{45}$$

where γ is the real market interest rate, ρ the rate of pure time preference of consumption, $\varepsilon > 0$ the elasticity of marginal consumption (income) and δ the consumption (income) growth rate. Clearly, the subjective element is the quantity ρ.With $\rho \approx 0.03$ and $\delta \approx 0.02$ as well as $\varepsilon = 1$ Nordhaus [26] obtains $\gamma \approx 0.05$. Arrow [2] estimates $\gamma \approx 0.03$ assuming $\rho \approx 0.01$, $\delta \approx 0.012$ and $\varepsilon = 1.5$ (!), however with tendency to larger values. In many other studies for sustainable development discount rates γ cluster around 5%. All those values are close to the real market rates or only a little smaller. Solow [42] , who presumes $\rho \approx 0.01$ to 0.02 adds a convergence condition for the (infinite) utility integral

$$\rho + \varepsilon\delta > n + \delta \tag{46}$$

to eq. (45). However, there are many authors in economics as well as philosophical and political sciences including Ramsey who refuse convincingly to accept a rate $\rho > 0$ in intergenerational contexts on ethical grounds ([40] [8] [32]) while it is considered fully acceptable for intragenerational discounting. On the other hand, intergenerational equity arguments in Arrow [2] indicate that there should be $\rho > 0$ in order to remove an "*... incredible and unacceptable strain on the present generation* ". Rabl [32] , who sets $\rho = 0$, argues that there must be $0 < \gamma < \varepsilon\delta$ in the framework of long-term public investments. Rabl neglects demographic aspects. As noted earlier we must have $\rho > n$ and, therefore, with $\rho \approx n$ at least $0 < \gamma < n + \varepsilon\delta$. On the basis of the Solow condition [42] one can, in fact, justify a rate ρ even slightly larger than n. One derives:

$$n + \delta(1-\varepsilon) < \rho < \gamma \le \gamma_{\max} < \beta = n + \varepsilon\delta \quad \text{or} \quad \beta = n + \delta \tag{47}$$

Values for ρ and β are presented in table 3. It is then possible to compute $\gamma_{\max} < \beta$ from eq. (7). $\gamma_{\max}$ usually is only insignificantly (1 to 20%) smaller than β depending on the specific case at hand, i.e. the particular sensitivities of $C(\mathbf{p})$ and $h(\mathbf{p})$ with respect to $\mathbf{p}$. The interest rates $\gamma_{\max}$ implied by the value of β are considerably lower, around 1.9%, than the usual real market interest rates. The above considerations based on a simple, ideal, steady state Ramseyan growth model in a closed economy can at least define the range of benefit and interest rates as well as reasonable rates of pure time preference to be used in long-term investments into life saving operations. It is believed that the steady state assumption of the Ramseyan model is not too far from reality in developed countries. Also, the assumption of an infinite time horizon is consistent with our general setting. Historical long-term population and economic growth rates cannot be questioned but there is considerable uncertainty about the future taking account of sustainability aspects. The value of ε varies very little, say between 0.75 and 0.85. Only the pure time preference rate ρ to be used in eq. (21) and possibly in eq. (39) can be subject to discussion and choice. It is suggested to take the lowest possible value which is $\rho = n + \delta(\mathbf{1} - \varepsilon)$ (see next paragraph). Of course, our considerations do not exclude larger rates of the time preference of consumption in special projects if there are no potential intergenerational conflicts. In the literature the adequacy of the Ramseyan model is sometimes questioned. For example, so-called overlapping generation models or generation adjusted discounting models are advocated instead. The main idea is to discount for living generations at the rate in eq. (47) with $\rho > 0$ but diminish the rate for all yet unborn generations down to n or even lower, thus facilitating the transition into a sustainable state of economy [32] [6]. But it is not expected that those refinements change our results significantly. Some further precautionary remarks are in order. The main body of environmental and economics literature on sustainable development agrees that economic growth will not persist, at least not at the long-term historical level. Natural resources will be depleted and arable land will become scarce. Many raise serious doubts whether the foreseeable demographic changes (aging populations and negative population growth in industrial countries) and the increasing scarcity of non-renewable natural resources and other environmental concerns can be compensated by technological progress. Optimists, on the other hand, are confident that technology will provide solutions. It is hard to predict what will actually happen. But there is an important mathematical result which may guide our choice. Weitzman [53] and others showed that the far-distant future should be discounted at the lowest possible rate ≥ 0 if there are different possible scenarios each with a given probability of being true. Exactly this strategy has been pursued in the foregoing. It should be noted that lowest possible interest rates so far have been chosen only for the subjective part ρ of the real interest rate γ. If reliable predictions of n and δ become available Weitzman's result should apply to all components of γ. The other somewhat subjective parameter ε in eq. (46) can as well be set at unity (corresponding to $u(c) = ln(c)$ implying $q \rightarrow 0$) and almost the same results will be obtained. Finally, it must be remembered that these rates should be used only when setting safety standards in the various fields, when investing into public health care programs, etc., by eqs. (38) or (39). They have very little to do with the rates the owner, the operator or the user would have to acquire from the financial market and which must be used when optimizing technical facilities with objectives (with or without life saving cost eq. (37) included) of the type eq. (3).

It is obvious that the results about the appropriate public interest rate for long-term investments are not yet fully conclusive and still controversial. More research and discussion is necessary.

7 Discussion and some results

Table 3 collects some relevant data for countries for which sufficiently reliable economic and demographic data are available. The data can vary depending on the type and date of the sources used. The life tables are all recent period life tables of different length from national statistical offices or from [7] . n, m, e and w are taken at their present values but slow demographic changes could, in principle, be considered. The age distribution $h(a, n)$ in table 3 is determined from recent period life tables. A stable population is assumed. Because the largest age a_u considered in the life tables is around 110 years this is also the time span over which our considerations are valid. The economic growth rate δ has been averaged over the years 1870 to 1992. It certainly would be misleading to take only averages over the last 50 years or less. The values for ρ, β, $SLSC$, $G_{\pi\overline{E}}$, $G_{\Delta\overline{E}}$ and $\overline{SVSL}$ are then calculated from these data using eqs. (47), 34 and (27). The demographic constants $G_{x\overline{E}}$ can be calculated by multiplying the value given in table 3 by the corresponding q/g. The largest uncertainties are possibly due to the part of GDP effectively available for risk reduction and due to the life working time estimates. As suggested earlier the part of the GDP available for risk reduction is taken as that available for private use. The Scandinavian countries have comparatively low values due to a smaller share g of their GDP for private use in the official sources yielding too small values of $G_{\pi\overline{E}}$, $G_{\Delta\overline{E}}$ or $\overline{SVSL}$. Some adjustments are necessary so that the quantity g really includes all what is available for risk reduction. The Netherlands and, in part, also Norway are exceptional in that recent sources give a rather low q (possibly due to a large proportion of part time employment but also due to the fact that the statistics contain only dependent employment). Factors like the unemployment rate, the productivity level of the labor force and the specific legal and social system must also be considered. The relatively high value of q for the USA appears partially to be due to the household survey technique as opposed to the establishment survey technique used in most other countries [28] [29]. Although the work-leisure principle outlined before may still be valid in general it appears that the accounting of life working time must be improved for our purposes. The influence of the mortality regime of the risk reducing measure is remarkable for all countries. Some countries have almost zero or even negative population growth rates and, consequently, very small ρ and β Although the best possible has been made out of the available data, some uncertainties and ambiguities remain, mainly due to differences in the way statistical data are taken in different countries. The results cover most industrialized countries including some extremes. They show the complex interaction of past and present economic conditions with demographic factors. They should be considered as preliminary estimates, especially if one wishes to compare across countries. If one excludes the Eastern European countries and tentatively adjusts for the deficiencies of certain data the values *SLSC*, $G_{\pi\overline{E}}$, $G_{\Delta\overline{E}}$ and $\overline{SVSL}$ for a number of countries are surprisingly close together. They are sufficiently close together for most practical applications.

The estimates for $G_{x\overline{E}}(\zeta, \rho, n)$ and, to a lesser degree, $\overline{SVSL}$ are in good agreement with several other estimates in the literature based on various, by far and large empirical concepts such as compensating wage differentials in the labor market and contingent valuation studies (see, for example, [46] [50] [23] [52] [4] [1] and many others). Most of those estimates are between 100,000 and over 10 Mill. PPP US$ with a clustering around 5 Mill. PPP US$. The agreement is especially good if no age-averaging is performed in the theoretical calculations and / or the full GDP is used.

For public risk reduction interventions the interest rates, i.e. ρ as well as γ to be inferred from β, shown in table 3 appear low enough to be acceptable, especially in view of the large uncertainties when assessing the quantities k and N_{PE} in eq. (37) for (3) or (39). Note that larger β's tend to occur whenever the population and / or economic growth rates are also larger. In general, the β's are smaller than presently used for public investments which are between 2 and 7 %. Further, from table 3 one observes $\gamma \leq \beta \approx \delta + n$ implying that discounting on both sides of eq. (39) has little effect, i.e. $\frac{\delta}{\gamma}$ is a little smaller but close to unity.

Table 3: Social indicators for some countries: [1)] in PPPUS\$, [2)] private consumption in PPPUS\$ according to [48], [3)] economic growth in % for 1870-1992 after [27], [4)] crude mortality (2000) in % [10], [5)] population growth (2000) in % [10], [6)] estimates based on [17] [27] [12] including 1 hour travel time per working day and a life working time of 45 years, [7)] SLSC computed with g and age-averaged life expectancies

Country	GDP [1)], g [2)]	δ [3)]	m [4)]	n [5)]	e	q [6)]	ρ	β	*SLSC* [7)]	$G_{\pi\overline{E}}$, $G_{\Delta\overline{E}}$	*SVSL*
Canada	27330, 16040	2.0	0.73	0.99	78	0.15	1.6	3.0	$7.0 \cdot 10^5$	$1.7 \cdot 10^6$, $1.9 \cdot 10^6$	$2.2 \cdot 10^6$
USA	34260, 22030	1.8	0.87	0.90	77	0.17	1.5	2.7	$9.1 \cdot 10^5$	$2.1 \cdot 10^6$, $2.3 \cdot 10^6$	$2.7 \cdot 10^6$
Austria	26310, 14790	1.8	0.98	0.24	77	0.13	0.6	2.0	$5.9 \cdot 10^5$	$1.7 \cdot 10^6$, $2.3 \cdot 10^6$	$2.8 \cdot 10^6$
Bulgaria	6200, 4400	1.3	1.45	-1.14	70	0.17	-0.9	0.2	$1.3 \cdot 10^5$	$4.6 \cdot 10^5$, $5.6 \cdot 10^5$	$7.5 \cdot 10^5$
Czech Rep.	12900, 6730	1.5	1.08	-0.07	73	0.17	0.3	1.4	$2.3 \cdot 10^5$	$5.8 \cdot 10^5$, $7.0 \cdot 10^5$	$8.8 \cdot 10^5$
Denmark	25500, 12900	1.8	1.09	0.30	77	0.12	0.6	2.1	$5.2 \cdot 10^5$	$1.5 \cdot 10^6$, $2.0 \cdot 10^6$	$2.4 \cdot 10^6$
Finland	22900, 12100	1.8	0.98	0.16	77	0.15	0.6	2.3	$4.7 \cdot 10^5$	$1.2 \cdot 10^6$, $1.5 \cdot 10^6$	$1.8 \cdot 10^6$
France	24470, 14660	1.9	0.91	0.37	78	0.13	0.8	2.3	$6.0 \cdot 10^5$	$1.8 \cdot 10^6$, $2.1 \cdot 10^6$	$2.5 \cdot 10^6$
Germany	25010, 14460	1.9	1.04	0.27	78	0.13	0.6	2.2	$5.8 \cdot 10^5$	$1.7 \cdot 10^6$, $2.1 \cdot 10^6$	$2.6 \cdot 10^6$
Ireland	25470, 12610	1.5	0.81	1.12	76	0.15	1.6	2.6	$5.4 \cdot 10^5$	$1.3 \cdot 10^6$, $1.6 \cdot 10^6$	$2.0 \cdot 10^6$
Italy	23400, 14460	1.9	1.01	0.07	79	0.14	0.4	2.0	$5.7 \cdot 10^5$	$1.6 \cdot 10^6$, $2.1 \cdot 10^6$	$2.5 \cdot 10^6$
Hungary	11200, 6870	1.2	1.32	-0.32	71	0.17	-0.1	0.9	$2.3 \cdot 10^5$	$7.3 \cdot 10^5$, $8.7 \cdot 10^5$	$1.1 \cdot 10^6$
Netherlands	26170, 15470	1.5	0.87	0.55	78	0.11	0.8	2.1	$6.6 \cdot 10^5$	$2.3 \cdot 10^6$, $2.9 \cdot 10^6$	$3.6 \cdot 10^6$
Norway	29760, 14149	2.1	0.98	0.49	78	0.11	0.9	2.6	$6.0 \cdot 10^5$	$1.6 \cdot 10^6$, $2.3 \cdot 10^6$	$2.7 \cdot 10^6$
Poland	9030, 5630	1.6	1.00	-0.03	73	0.17	0.3	1.6	$2.0 \cdot 10^5$	$6.6 \cdot 10^5$, $6.8 \cdot 10^5$	$8.5 \cdot 10^5$
Russia	8377, 5440	1.2	1.34	-0.35	66	0.18	-0.2	0.9	$1.7 \cdot 10^5$	$5.8 \cdot 10^5$, $6.1 \cdot 10^5$	$7.9 \cdot 10^5$
Sweden	23770, 12620	1.9	1.06	0.02	79	0.14	0.4	1.9	$5.0 \cdot 10^5$	$1.3 \cdot 10^6$, $1.8 \cdot 10^6$	$2.2 \cdot 10^6$
Switzerland	29000, 17700	1.9	0.88	0.27	79	0.13	0.8	2.2	$7.3 \cdot 10^5$	$2.1 \cdot 10^6$, $2.6 \cdot 10^6$	$3.1 \cdot 10^6$
UK	23500, 15140	1.3	1.07	0.23	78	0.15	0.5	1.5	$5.9 \cdot 10^5$	$1.6 \cdot 10^6$, $2.1 \cdot 10^6$	$2.8 \cdot 10^6$
Japan	26460, 15960	2.7	0.83	0.17	80	0.15	0.6	2.9	$6.4 \cdot 10^5$	$1.6 \cdot 10^6$, $1.8 \cdot 10^6$	$2.0 \cdot 10^6$
Australia	25370, 15750	1.2	0.72	0.99	78	0.16	1.2	2.2	$6.8 \cdot 10^5$	$1.8 \cdot 10^6$, $2.0 \cdot 10^6$	$2.4 \cdot 10^6$

8 Illustration example

As an example from the structures area we take a rather simple case of a single-mode system where failure is defined if a random resistance or capacity is exceeded by a random demand. The demand is modelled as a one-dimensional, stationary marked Poissonian renewal process of disturbances (earthquakes, wind storms, explosions, etc.) with stationary renewal rate λ and random, independent sizes of the disturbances S_i, i = 1, 2, ….. The resistance is log-normally distributed with mean p and a coefficient of variation V_R. The disturbances are also independently log-normally distributed with mean equal to unity and coefficient of variation V_S

so that p can be interpreted as central safety factor. A disturbance causes failure with probability:

$$P_f(p) = \Phi\left(-\frac{\ln\left\{ p\sqrt{\dfrac{1+V_S^2}{1+V_R^2}} \right\}}{\sqrt{\ln\left((1+V_R^2)(1+V_S^2)\right)}} \right) \tag{48}$$

An appropriate objective function then is with $b = b(\mathbf{p})$:

$$Z(p) = \frac{b}{C_0\gamma} - \left(1 + \frac{C_1}{C_0} p^a\right) - \left(1 + \frac{C_1}{C_0} p^a + \frac{H_M}{C_0} + \frac{H_F}{C_0}\right) \frac{\lambda P_f(p)}{\gamma} \tag{49}$$

The criterion (39) has the form:

$$\frac{d}{dp}\left(C_0 + C_1 p^a\right) \geq -G_{x\overline{E}}(\zeta, \rho, n) k N_{PE} \frac{\delta}{\gamma} \frac{d}{dp}\left(\lambda P_f(p)\right) \tag{50}$$

Some more or less realistic, typical parameter assumptions are: $C_0 = 10^6$, $C_1 = 10^4$, $a = 1.25$, $H_M = 3 \cdot C_0$, $V_R = 0.2$, $V_S = 0.3$, and $\lambda = 1$ [1 / *year*]. The *LQI*-data is $e = 77$, $GDP = 25{,}000$, $g = 15{,}000$, $m = 0.01$, $C_{xE} = 25$, $w = 0.125$, $N_{PE} = 100$, $k = 0.1$ so that $H_F = SLSC\ kN_{PE} = 5.4 \cdot 10^6$, $G_{x\overline{E}}(\zeta, \rho, n) = 2.6 \cdot 10^7$.

The value of N_{PE} is chosen relatively large for demonstration purposes. Monetary values are in US$. Optimization is performed for the public and for the owner separately.

For the public b_s=0.032 and γ_s=0.03 are chosen. Also, we take $\frac{\delta}{\gamma_S} = 1$ for simplicity.

In particular, benefit and discount rate are chosen such that the public does not make direct profit from an economic activity of its members. Optimization including the cost H_F gives $p_S^* = 4.21$, the corresponding failure rate is $2.0 \cdot 10^{-5}$. Criterion (39) is already fulfilled for p_{lim} = 3.3 corresponding to a yearly failure rate of $2.5 \cdot 10^{-4}$, but $Z_S(p_{lim}) / C_0$ being already negative. It is interesting to see that in this case the public can do better in adopting the optimal solution rather than just realizing the facility at its acceptability limit as pointed out already in section 6. The owner uses some typical values of $b_O = 0.07 C_0$ and $\gamma_O = 0.05$ and does or does not include life saving cost. If he includes life saving cost the objective function is shifted to the right (dotted line). The calculations yield $p_O^* = 3.76$ and $p_O^* = 4.03$, respectively, and the corresponding failure rates are $7.1 \cdot 10^{-5}$ and $3.2 \cdot 10^{-5}$.The LQI-based acceptability criterion limits the owner's region for reasonable designs. Inclusion of life saving cost has relatively little influence on the position of the optimum.

It is noted that the stochastic model and the variability of capacity and demand also play an important role for the magnitude and location of the optimum as well as the acceptability limit.

The specific marginal cost (rate of change) of a safety measure and its effect on a reduction of the failure rate are equally important.

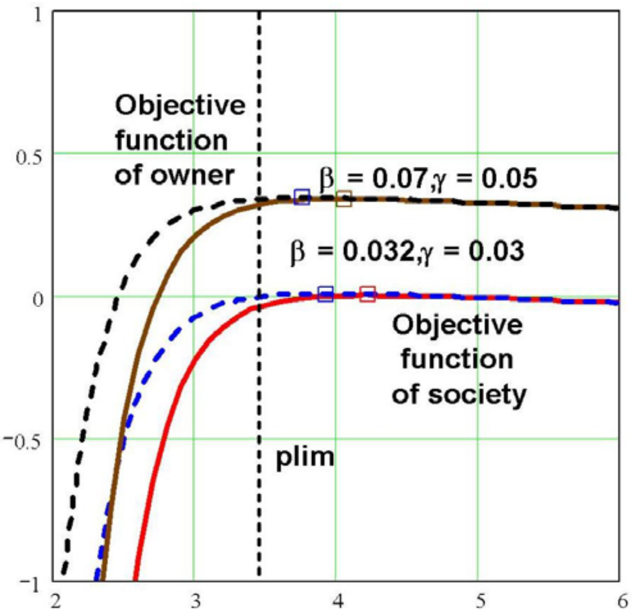

Figure 1: Objective function for society and owner with (solid lines) and without (dashed lines) life saving cost)

This example also allows to derive risk-consequence curves by varying the number of fatalities in an event. With the same data as before but $SLSC = 10^6$ and $G_{x\overline{E}}(\rho,n) = 4 \cdot 10^6$ for $N_F = 1$ we first vary the cost effectiveness of the safety measure. Here, only the ratio C_1 / C_0 is changed. Most realistic is probably a ratio of $C_1 / C_0 = 0.001$ or less. The failure rate of approximately 10^{-4} per year corresponds well with the controllable crude mortality of the same magnitude as mentioned earlier. In figure 3 the mortality regimes are varied (see eq. (18) and (19)) indicating that this is of only moderate influence. In this figure the so-called ALARP-region (ALARP = **A**s **L**ow **A**s **R**easonably **P**racticable) is also shown.
Note that in these figures the failure rate is given by λP_f and the number of fatalities is given by $N_F = kN_{PE}$. Therefore, these figures cover for the full range of λ and P_f and k and N_{PE}, respectively. However, it is to be mentioned again that in both figures the precise location and slope of the acceptability curve depend on the specific physical and stochastic model.

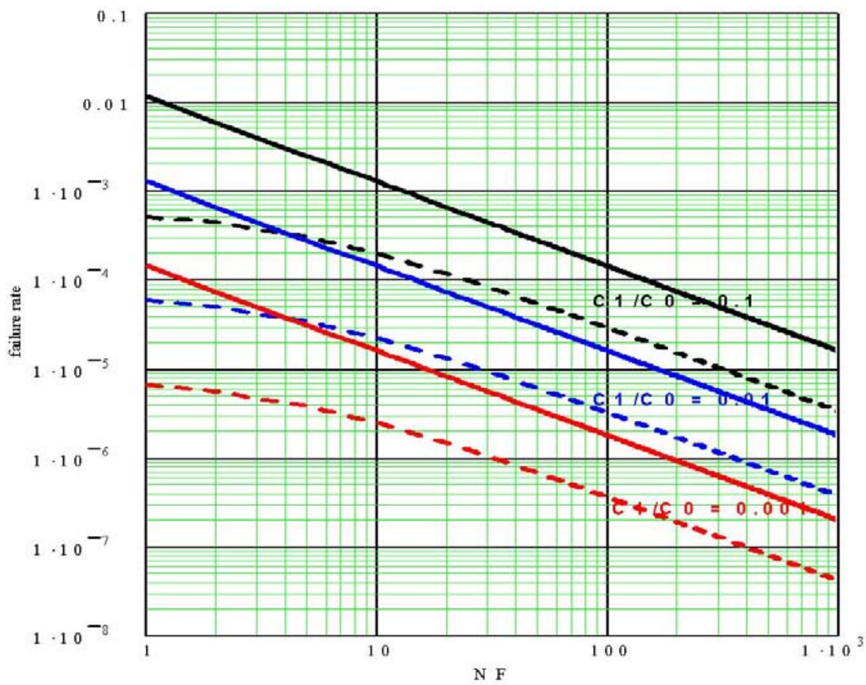

Figure 2: Acceptable Failure rate over number of fatalities for different C_1.Dashed lines correspond to optimal solution for the public.

9 Summary and conclusions

Technical facilities should be optimal with respect to benefits and cost. The risk of failure and especially the risk to human life and limb must be limited. A suitable objective function is developed based on a renewal model. The *life quality index* is presented and embedded into modern socioeconomic concepts. A general risk acceptability criterion is derived. Most importantly, the societal *SLSC* (societal life saving cost = implied cost of averting a fatality) to be used in optimization as live saving or compensation cost and the *societal willingness-to-pay* based on the *societal value of a statistical life* or on the *societal life quality index* are derived, the latter for two different regimes to reduce mortality. The acceptability criterion, which is *necessary*, *affordable* and *efficient* from a societal point of view, depends on the marginal cost to reduce the risk, the corresponding marginal decrease in risk, the GDP, the life working time and on demographic factors obtainable from life tables. For example, key parameters such as the societal life saving cost (*SLSC*) and societal value of a statistical life (*SVSL*) turn out to cluster around 600,000 PPPUS$ and 2.5 Mill. PPPUS$, respectively, with very little variation for industrialized countries. Because future risks are considered most of the time-dependent factors have been investigated and taken into account up to a time horizon of approximately 100 years. It appears remarkable that cost-benefit optimal facilities usually provide more safety than the acceptability criterion in most cases if life saving cost are included in the analysis.

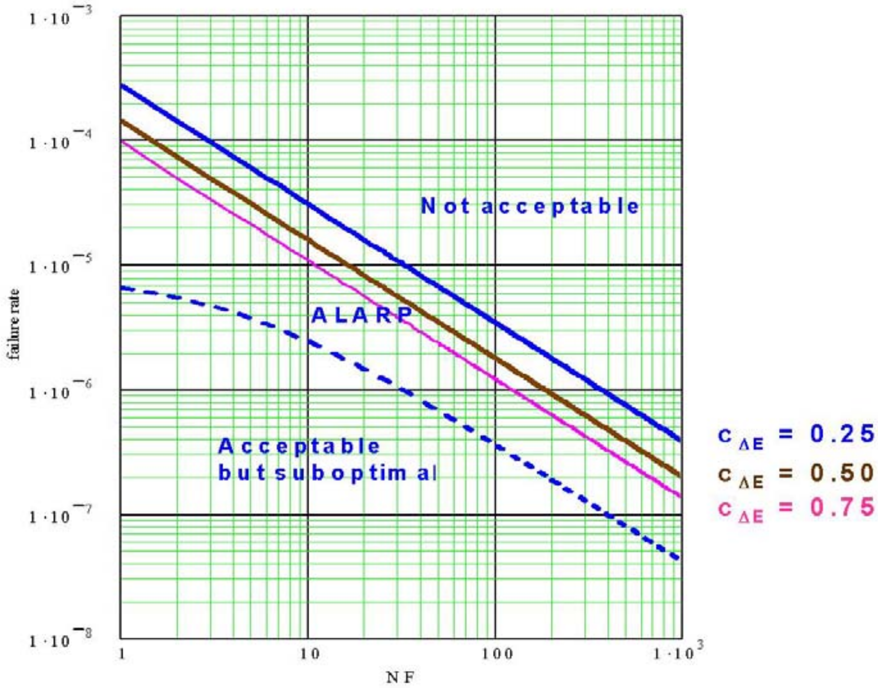

Figure 3: Acceptable risk for different mortality regimes.

If time is involved all monetary quantities need to be discounted down to the decision point. Discount rates γ must be long term averages in view of the time horizon of some 20 to more than 100 years for the facilities of interest and net of inflation and taxes. While the operator may use long term averages from the financial market for his cost-benefit analysis the assessment of interest rates for investments of the public into risk reduction is more difficult. The classical Ramsey model decomposes the output growth rate into the rate of time preference of consumption and the rate of economical growth multiplied by the elasticity of marginal utility of consumption. It is found that the rate of time preference of consumption should be a little larger than the long term population growth rate if used for the determination of parameters in the acceptability criterion. The output growth rate (= public interest rate) on the other hand should be smaller than the sum of the population growth rate and the long term growth rate of a national economy which is around 2% for most industrial countries. Accordingly, the rate of time preference of consumption is about 1% or less, which is also intergenerationally acceptable from an ethical point of view. All in all, discounting plays an important role in public cost-benefit considerations but is less important for a public risk acceptability criterion. It is also shown that given a certain output growth rate there is a corresponding maximum real interest rate in order to maintain non-negativity of the objective function in the public's interest. Technical and natural risks are perceived individually and on a societal level rather irrationally and subjectively. Frequently, risks are communicated to and from the public in such a way that regulatory bodies or other authorities rarely can apprehend fully the nature, magnitude and severity of specific, recognized risks. Accordingly, they hardly are in a position to respond rationally by efficient risk control measures. A lack of efficiency has been shown in a number of studies, among others in the study of [46] , indicating that many if not most public risk reduction interventions are highly inefficient. Some others can be shown to be, in fact, no more affordable thus taking away resources needed for other risk reducing projects and / or reducing life quality in the sense that other components of life quality than life expectancy are inedaquately diminuished. A third group of risks is inadequately taken into account because the benefits from an undertaking appear to be overwhelming. Only if society acts rationally in controlling involuntary and anonymous risks from the natural and technical environment in an affordable and efficient manner can society gain better life quality in the long run.

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